

# Coexistence of antiferromagnetism and $d$ -wave singlet state controlled by long-range hopping integral

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Interplay between antiferromagnetism (AF) and  $d$ -wave superconductivity ( $d$ SC) is investigated in the slave-boson scheme of the two-dimensional  $t$ - $J$  model on the square lattice. So far, it seems that their coexistence is believed to be a general feature. It is, however, reported in this paper that the coexistence is suppressed significantly by  $t''$ , the third neighbor hopping. This effect will lead to noticeable material dependence of the possible bulk coexistence of AF and  $d$ SC.

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## I. INTRODUCTION

Interplay between antiferromagnetism (AF) and  $d$ -wave superconductivity ( $d$ SC) is one of the most interesting issues in high- $T_c$  cuprates. In particular, it is a fundamental question whether or not the bulk coexistence of AF and  $d$ SC is possible. The theoretical studies on the two-dimensional (2D)  $t$ - $J$  model[1–4] and extended Hubbard models,[5–8] which are believed minimal for the description of high- $T_c$  cuprates, predict that it is possible. This prediction independent of models might imply a general possibility of bulk coexistence in high- $T_c$  cuprates. However, such a possibility is reported only in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO) systems,[9] and seems to have strong material dependence.

In this paper, we study the material dependence of the coexistence of AF and  $d$ SC in the 2D  $t$ - $J$  model, specifically, the dependence on the second ( $t'$ ) and third ( $t''$ ) neighbor hopping integrals. Focus is put on several values around the realistic ones:  $t'/t = -1/6$  and  $t''/t = 0$  for LSCO systems, and  $t'/t = -1/6$  and  $t''/t = 1/5$  for  $\text{YBa}_2\text{Cu}_3\text{O}_{6+y}$  (YBCO) systems.[10–12] We find that without  $t''$ , the coexistence is realized in a wide doping region, in accordance with the previous work[3]. However, once  $t''$  is introduced, the coexistence is suppressed significantly. This effect of  $t''$  is shown by investigating two routes into the coexistence, namely, the  $d$ SC instability in the (metallic) AF state, and the AF instability in the  $d$ SC state. We discuss the generality of the present finding, and implications for actual systems.

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## II. MODEL AND FORMALISM

We analyze the 2D  $t$ - $J$  model on the square lattice,

$$H = - \sum_{i,j,\sigma} t^{(l)} \tilde{c}_{i\sigma}^\dagger \tilde{c}_{j\sigma} + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

defined in the Fock space with no doubly occupied sites. Here  $\tilde{c}_{i\sigma}$  ( $\mathbf{S}_i$ ) is an electron (a spin) operator. The  $t^{(l)}$  is the  $l$ th neighbor hopping integral, and we denote  $t^{(1)} = t$ ,  $t^{(2)} = t'$  and  $t^{(3)} = t''$ . The  $J(>0)$  is the superexchange coupling between nearest-neighbor sites. We adopt the slave-boson mean-field scheme by writing  $\tilde{c}_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i$ , where  $f_{i\sigma}$  ( $b_i$ ) is a fermion (boson) operator that carries spin  $\sigma$  (charge  $e$ ),  $\mathbf{S}_i = \sum_{\alpha\beta} \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}$ , with Pauli matrix  $\boldsymbol{\sigma}$ , and introducing the following mean fields: for AF,  $m \equiv \frac{1}{2} \langle \sum_{\sigma} \sigma f_{i\sigma}^\dagger f_{i\sigma} \rangle e^{i\mathbf{Q} \cdot \mathbf{r}_i}$ ,  $\mathbf{Q} = (\pi, \pi)$ , and for resonating valence bond (RVB),  $\chi^{(l)} \equiv \langle \sum_{\sigma} f_{i\sigma}^\dagger f_{j\sigma} \rangle$ ,  $\langle b_i^\dagger b_j \rangle$  and  $\Delta_\tau \equiv \langle f_{i\uparrow} f_{i+\tau\downarrow} - f_{i\downarrow} f_{i+\tau\uparrow} \rangle$ ,  $\tau = x, y$ . These mean fields are taken to be real constants independent of sites  $i$  and  $j$ . The  $d$ -wave symmetry  $\Delta_0 \equiv \Delta_x = -\Delta_y \neq 0$  is stable at low  $T$  and this state is called the  $d$ -wave singlet RVB ( $d$ RVB). The  $d$ SC state is defined as  $\Delta_0 \neq 0$  and  $\langle b \rangle \neq 0$ . In the following, we assume  $\langle b \rangle = \sqrt{\delta}$  where  $\delta$  is the hole density, and focus on the fermion part; the  $d$ SC is then associated directly with the  $d$ RVB. This assumption is valid at low  $T$  and for  $\delta$  not close to half filling ( $\delta \gtrsim 0.02$ ).[3] The mean-field Hamiltonian is given by

$$H_{\text{MF}} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \begin{pmatrix} \xi_{\mathbf{k}} & -\Delta_{\mathbf{k}} & -2Jm & 0 \\ -\Delta_{\mathbf{k}} & -\xi_{\mathbf{k}} & 0 & -2Jm \\ -2Jm & 0 & \xi_{\mathbf{k}+\mathbf{Q}} & -\Delta_{\mathbf{k}+\mathbf{Q}} \\ 0 & -2Jm & -\Delta_{\mathbf{k}+\mathbf{Q}} & -\xi_{\mathbf{k}+\mathbf{Q}} \end{pmatrix} \Psi_{\mathbf{k}}, \quad (2)$$

with a global constraint  $\sum_{\sigma} \langle f_{i\sigma}^{\dagger} f_{i\sigma} \rangle = 1 - \delta$ . The  $\mathbf{k}$  sum is over the magnetic Brillouin zone  $|k_x| + |k_y| \leq \pi$  and

$$\Psi_{\mathbf{k}}^{\dagger} = \left( f_{\mathbf{k}\uparrow}^{\dagger} \quad f_{-\mathbf{k}\downarrow} \quad f_{\mathbf{k}+\mathbf{Q}\uparrow}^{\dagger} \quad f_{-\mathbf{k}+\mathbf{Q}\downarrow} \right), \quad (3)$$

$$\xi_{\mathbf{k}} = -2 \left[ \left( t\delta + \frac{3}{8} J\chi^{(1)} \right) (\cos k_x + \cos k_y) + 2t'\delta \cos k_x \cos k_y + t''\delta (\cos 2k_x + \cos 2k_y) \right] - \mu, \quad (4)$$

$$\Delta_{\mathbf{k}} = -\frac{3}{4} J\Delta_0 (\cos k_x - \cos k_y), \quad (5)$$

with  $\mu$  being the chemical potential. The mean fields are determined by solving the following self-consistent equations numerically:

$$m = \frac{1}{N} \sum_{\mathbf{k}} \frac{Jm}{D_{\mathbf{k}}} \left( \frac{\eta_{\mathbf{k}}^+}{\lambda_{\mathbf{k}}^+} \tanh \frac{\lambda_{\mathbf{k}}^+}{2T} - \frac{\eta_{\mathbf{k}}^-}{\lambda_{\mathbf{k}}^-} \tanh \frac{\lambda_{\mathbf{k}}^-}{2T} \right), \quad (6)$$

$$\chi^{(1)} = -\frac{1}{2N} \sum_{\mathbf{k}} \frac{\xi_{\mathbf{k}}^-}{D_{\mathbf{k}}} (\cos k_x + \cos k_y) \times \left( \frac{\eta_{\mathbf{k}}^+}{\lambda_{\mathbf{k}}^+} \tanh \frac{\lambda_{\mathbf{k}}^+}{2T} - \frac{\eta_{\mathbf{k}}^-}{\lambda_{\mathbf{k}}^-} \tanh \frac{\lambda_{\mathbf{k}}^-}{2T} \right), \quad (7)$$

$$\Delta_0 = -\frac{1}{2N} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) \times \left( \frac{\Delta_{\mathbf{k}}}{\lambda_{\mathbf{k}}^+} \tanh \frac{\lambda_{\mathbf{k}}^+}{2T} + \frac{\Delta_{\mathbf{k}}}{\lambda_{\mathbf{k}}^-} \tanh \frac{\lambda_{\mathbf{k}}^-}{2T} \right), \quad (8)$$

$$\delta = \frac{1}{N} \sum_{\mathbf{k}} \left( \frac{\eta_{\mathbf{k}}^+}{\lambda_{\mathbf{k}}^+} \tanh \frac{\lambda_{\mathbf{k}}^+}{2T} + \frac{\eta_{\mathbf{k}}^-}{\lambda_{\mathbf{k}}^-} \tanh \frac{\lambda_{\mathbf{k}}^-}{2T} \right). \quad (9)$$

Here  $\lambda_{\mathbf{k}}^{\pm} = \sqrt{\eta_{\mathbf{k}}^{\pm 2} + \Delta_{\mathbf{k}}^2}$  is the quasiparticle energy in the coexistent state,  $\eta_{\mathbf{k}}^{\pm} = \xi_{\mathbf{k}}^{\pm} \pm D_{\mathbf{k}}$  is that in the AF state,

$D_{\mathbf{k}} = \sqrt{(\xi_{\mathbf{k}}^-)^2 + (2Jm)^2}$ ,  $\xi_{\mathbf{k}}^{\pm} = (\xi_{\mathbf{k}} \pm \xi_{\mathbf{k}+\mathbf{Q}})/2$ , and  $T$  ( $N$ ) is temperature (the total number of lattice sites).

### III. RESULTS

Figure 1(a) shows the phase diagram on the plane of  $T$  versus  $\delta$  for the band parameter,  $t/J = 4$ ,  $t'/t = -1/6$ , and  $t''/t = 0$ , which will be appropriate to LSCO.[13–15] The  $T_N$  is the onset temperature of AF, whereas  $T_{\text{RVB}}^{\text{AF}}$  ( $T_{\text{RVB}}$  and  $T_{\text{RVB}}^{\text{noAF}}$ ) is that of  $d$ RVB in the presence (absence) of AF. The (commensurate) AF phase is stabilized in a wide doping region,  $\delta \lesssim \delta_N = 0.159$ , where  $\delta_N$  is a critical doping rate of AF ordering at  $T = 0$ , and suppresses the  $d$ RVB instability ( $T_{\text{RVB}}^{\text{AF}} < T_{\text{RVB}}^{\text{noAF}}$ ). These features are already seen in the early work.[3] The  $\delta$  dependence of the order parameters is shown in Fig. 1(b). With decreasing  $\delta$ , the AF is realized through a second-order transition and it suppresses  $\Delta_0$  and  $\chi^{(1)}$ . Figure 1(c) shows

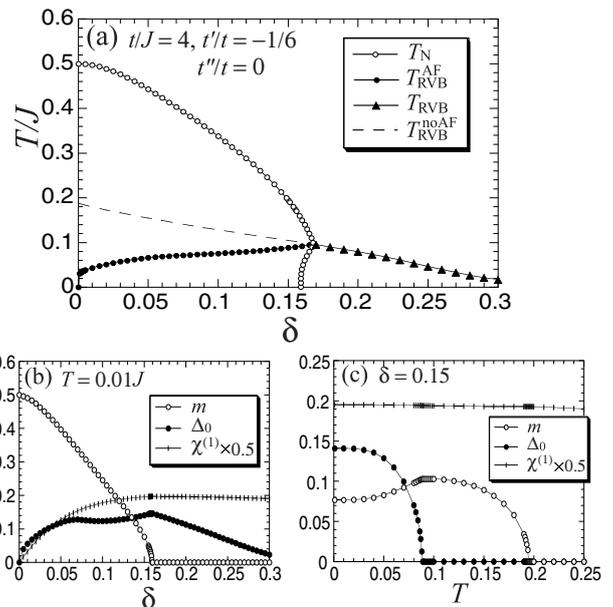


FIG. 1: (a) The phase diagram on the plane of  $T$  and  $\delta$  for  $t/J = 4$ ,  $t'/t = -1/6$  and  $t''/t = 0$ . The  $T_N$  is the onset temperature of AF, and  $T_{\text{RVB}}^{\text{AF}}$  ( $T_{\text{RVB}}$  and  $T_{\text{RVB}}^{\text{noAF}}$ ) is that of  $d$ RVB in the presence (absence) of AF. (b)  $\delta$  dependence and (c)  $T$  dependence of the order parameters at  $T = 0.01J$  and  $\delta = 0.15$ , respectively.

the  $T$  dependence of the order parameters. Both the AF and the  $d$ RVB are realized through a second-order transition, and the  $d$ RVB ordering is accompanied by a small suppression of AF. (This suppression is not clear at low  $\delta$ , because of large  $m$ .) The change of  $\chi^{(1)}$  is negligible below  $T_N$  and  $T_{\text{RVB}}^{\text{AF}}$ , that is, the coherency of fermion's hopping is not disturbed appreciably.

The primary finding of the present study is a significant effect of  $t''$ . Figure 2(a) shows the phase diagram with the inclusion of  $t''/t = 0.2$ , which will be appropriate to YBCO. The phase diagram is qualitatively different from Fig. 1(a); the  $d$ RVB instability in the AF state is strongly suppressed in a range of moderate hole density. The order parameters, especially  $m$  and  $\Delta_0$ , also behave differently. Figure 2(b) shows that the AF is realized through a first-order-like transition as a function of  $\delta$ , accompanied by the rapid suppression of  $d$ RVB. As a function of  $T$ , on the other hand, we see in Fig. 2(c) that while the AF order develops continuously below (higher)  $T_N$ , it suddenly drops once the  $d$ RVB sets in through a nearly first-order transition. This is seen in the region close to  $\delta_N = 0.128$ . Away from this region,  $T$  dependence is similar to Fig. 1(c).

Why does  $t''$  have such significant effects and lead to the sharp contrast between Fig. 1 and Fig. 2? To under-

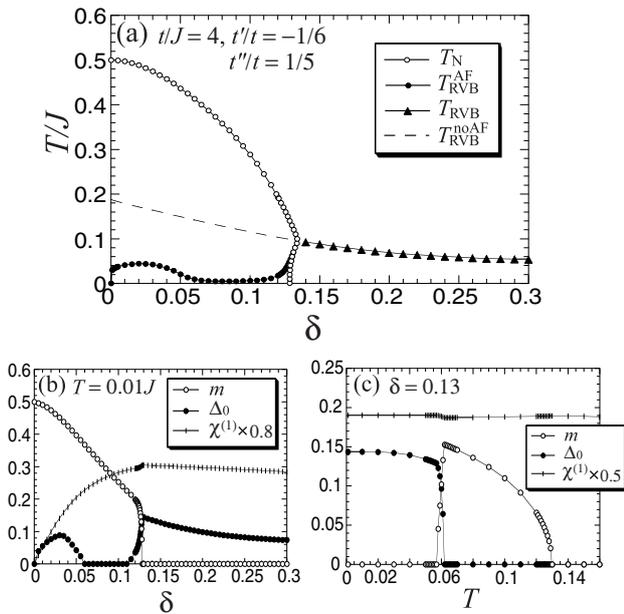


FIG. 2: (a)  $T$ - $\delta$  phase diagram for  $t/J = 4$ ,  $t'/t = -1/6$ , and  $t''/t = 1/5$ . (b)  $\delta$  dependence and (c)  $T$  dependence of the order parameters at  $T = 0.01J$  and  $\delta = 0.13$ , respectively.

stand this, we analyze the onset equation of  $d$ RVB,

$$T_c = \frac{3}{16N} \sum_{\mathbf{k}} (\cos k_x - \cos k_y)^2 \times \left( \frac{2T_c}{\eta_{\mathbf{k}}^+} \tanh \frac{\eta_{\mathbf{k}}^+}{2T_c} + \frac{2T_c}{\eta_{\mathbf{k}}^-} \tanh \frac{\eta_{\mathbf{k}}^-}{2T_c} \right). \quad (10)$$

Here  $T_c$  is  $T_{\text{RVB}}^{\text{AF}}$  for  $m \neq 0$  and  $T_{\text{RVB}}^{\text{noAF}}$  for  $m \equiv 0$ . The bands in the AF state  $\eta_{\mathbf{k}}^{\pm}$  are shown in Fig. 3(a) together with those for  $m \equiv 0$ . Since the  $\eta_{\mathbf{k}}^+$  is pushed up to high energy and only  $\eta_{\mathbf{k}}^-$  extends to low energy, the term with  $\eta_{\mathbf{k}}^-$  is relevant in Eq. (10). For  $m \equiv 0$ , on the other hand, both terms with  $\eta_{\mathbf{k}}^{\pm}$  contribute. Thus,  $T_{\text{RVB}}^{\text{AF}}$  is generally lowered from  $T_{\text{RVB}}^{\text{noAF}}$ . The degree of this suppression is controlled mainly by  $\eta^-(\pi, 0)$  because of the  $d$ -wave form factor in Eq. (10). To measure  $\eta^-(\pi, 0)$ , we consider the following quantity:

$$W(t', t'') \equiv [\eta^-(\pi/2, \pi/2) - \eta^-(\pi, 0)]/4\delta \quad (11)$$

$$= 2t'' - t'. \quad (12)$$

For the present parameter,  $t' < 0$  and  $t'' \geq 0$ ,  $W(t', t'')$  is positive. This means that the Fermi surface (FS) or the hole pocket is formed around  $(\pi/2, \pi/2)$  [Fig. 3(b)] independent of the values of  $t'$  and  $t''$ . [16] Since the area of the hole pocket is determined uniquely by  $\delta$ , the band parameter dependence of  $\eta^-(\pi/2, \pi/2)$  is weaker than that of  $\eta^-(\pi, 0)$  for a fixed  $\delta$ . Hence, the relative value of  $\eta^-(\pi, 0)$  among different band parameters will be measured by  $W(t', t'')$ . That is, the larger value of

$W(t', t'')$  means the larger magnitude of  $\eta^-(\pi, 0)$ , which suppresses  $T_{\text{RVB}}^{\text{AF}}$  more significantly [see Eq. (10)]. To demonstrate this explicitly, we first take  $t'' = 0$  and plot  $T_{\text{RVB}}^{\text{AF}}$  as a function of  $\delta/\delta_N (\leq 1)$  for several choices of  $t'$  in Fig. 4(a). As expected,  $T_{\text{RVB}}^{\text{AF}}$  is suppressed with increasing  $|t'|$  or  $W(t', t'')$ . The degree of the suppression depends on  $\delta/\delta_N$  and is most enhanced in a moderate doping region. This is because the AF order is not so strong near  $\delta/\delta_N \approx 1$  [see Fig. 1(b)], on one hand, and the hopping terms renormalization by  $\delta$  [see Eq. (4)] makes their effects ineffective at low  $\delta$ , on the other hand. The point is that compared with this  $t'$  effect,  $t''$  will have a much more significant effect, since  $t''$  has a prefactor 2 in Eq. (12) and the finite value of  $t''$  will generally imply a finite  $t'$ , which contributes to  $W(t', t'')$  additively. This is demonstrated in Fig. 4(b) by choosing several  $t''$ . We see a significant suppression even at small  $t''/t$ . (The recovery of  $T_{\text{RVB}}^{\text{AF}}$  in  $\delta/\delta_N \lesssim 0.4$  comes from the reduction of  $t'$  and  $t''$  effects at low  $\delta$  [see Eq. (4)].) It is to be noted that the present effect is a special feature of the band parameter,  $t' < 0$  and  $t'' \geq 0$ , where  $W(t', t'')$  is most enhanced. [16]

We have seen that  $t''$  is a crucial factor of the  $d$ RVB instability in the AF state. We next turn to the other side of the phase boundary, the AF instability in the  $d$ RVB state. This instability is determined by the condition  $\chi^{-1}(\mathbf{q}) = \chi_0^{-1}(\mathbf{q}) + 2J(\cos q_x + \cos q_y) = 0$ , where

$$\chi_0(\mathbf{q}) = \frac{1}{4N} \sum_{\mathbf{k}} \left[ C_{\mathbf{k}, \mathbf{k}+\mathbf{q}}^+ \frac{\tanh \frac{E_{\mathbf{k}}}{2T} - \tanh \frac{E_{\mathbf{k}+\mathbf{q}}}{2T}}{E_{\mathbf{k}} - E_{\mathbf{k}+\mathbf{q}}} + C_{\mathbf{k}, \mathbf{k}+\mathbf{q}}^- \frac{\tanh \frac{E_{\mathbf{k}}}{2T} + \tanh \frac{E_{\mathbf{k}+\mathbf{q}}}{2T}}{E_{\mathbf{k}} + E_{\mathbf{k}+\mathbf{q}}} \right], \quad (13)$$

$$C_{\mathbf{k}, \mathbf{k}+\mathbf{q}}^{\pm} = \frac{1}{2} \left( 1 \pm \frac{\xi_{\mathbf{k}} \xi_{\mathbf{k}+\mathbf{q}} + \Delta_{\mathbf{k}} \Delta_{\mathbf{k}+\mathbf{q}}}{E_{\mathbf{k}} E_{\mathbf{k}+\mathbf{q}}} \right), \quad (14)$$

and  $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$ . With mean fields determined under the constraint  $m \equiv 0$ , we calculate  $\chi_0(\mathbf{q})$  around  $(\pi, \pi)$  for several choices of  $t'$  and  $t''$ . To see the degree of suppression, we have scaled  $\chi_0(\mathbf{q})$  in Figs. 5(a) and 5(b) so that the peak height for  $t' = 0$  and for  $t'' = 0$  is unity, respectively. While  $\chi_0(\mathbf{q})$  is suppressed with increasing  $|t'|$  [Fig. 5(a)], we see much more significant suppression with  $t''$  [Fig. 5(b)]. This means that  $t''$  is crucial to suppress the AF instability in the  $d$ SC state. [We have checked that this qualitative feature does not depend on  $T$  and  $\delta (> 0)$ .]

Here we note effects of the underlying “FS” in the  $d$ RVB, which may be defined as  $E_{\mathbf{k}} = 0$  with  $\Delta_{\mathbf{k}} \equiv 0$ . For  $|t'/t| \lesssim 0.2$  at  $t''/t = 0$ , the “FS” is electronlike centered at the  $\Gamma$  point. The two clear incommensurate (IC) peaks of  $\chi_0(\mathbf{q})$  in Fig. 5(a) come from the nesting of the “FS.” For larger  $|t'/t|$  or with  $t''/t (\gtrsim 0.1)$ , the electronlike “FS” changes to a holelike “FS” centered at  $(\pi, \pi)$ , and loses the nesting property. Figure 5(b) shows that with

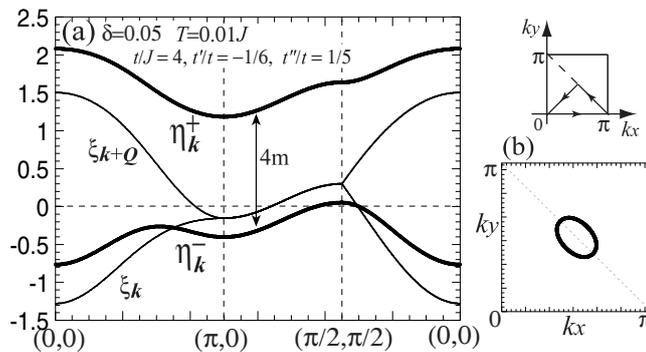


FIG. 3: (a) Energy band in the AF state (thick lines) and in the paramagnetic state with  $m \equiv 0$  (thin lines); the energy unit is  $J$ . The scanned path is shown in the upper right figure. (b) The Fermi surface in the AF state (thick line).

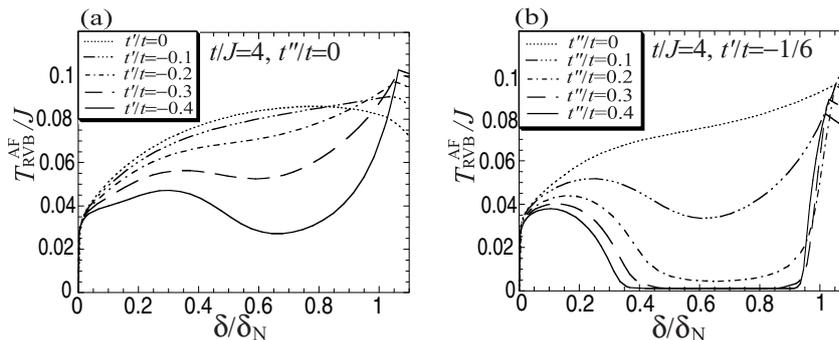


FIG. 4: The doping dependence of  $T_{\text{RVB}}^{\text{AF}}/J$  for several choices of  $t'$  (a) and  $t''$  (b). The  $\delta$  is scaled by  $\delta_{\text{N}}$ ;  $\delta_{\text{N}} = 0.151, 0.155, 0.161, 0.161, \text{ and } 0.147$  ( $0.159, 0.156, 0.128, 0.109, \text{ and } 0.095$ ) for  $t'/t = 0, -0.1, -0.2, -0.3, \text{ and } -0.4$  ( $t''/t = 0, 0.1, 0.2, 0.3, \text{ and } 0.4$ ) in (a)[(b)], respectively.

this “FS” being kept,  $\chi_0(\mathbf{q})$  is suppressed significantly by  $t''$ . That is, the topology of the “FS” is not relevant to the present finding.

#### IV. DISCUSSION AND CONCLUSION

We have shown that both the  $d\text{RVB}$  instability in the AF state (Figs. 1, 2, and 4) and the AF instability in the  $d\text{RVB}$  state (Fig. 5) are significantly suppressed by  $t''$ . Although this effect is found in the slave-boson mean-field scheme with the commensurate AF order, our finding will be rather general in the following senses.

(i) In the analysis of Eq. (11), the  $d\text{RVB}$  instability in the AF state is governed by the quasiparticle energy at  $\mathbf{k} = (\pi, 0)$ .

(ii) For the AF instability in the  $d\text{RVB}$  state, the suppression of  $\chi_0(\mathbf{q})$  by  $t''$  (Fig. 5) will be reflected in the full  $\chi(\mathbf{q})$  beyond the random phase approximation.

(iii) For the band parameter used in Fig. 1, the IC AF order will be more favorable at finite  $\delta$  and low  $T$ . [Note that  $\chi_0(\mathbf{q})$  shows the IC peaks in Fig. 5(b) for  $t'' = 0$ .] However, essential features may be captured by Fig. 1, since the calculation in the IC AF state[17] shows that the (segments of) FS is located near  $(\pi, 0)$  and  $(0, \pi)$ , which

will not severely block the scattering processes leading to the  $d\text{RVB}$  instability.

(iv) It is pointed out theoretically[7, 8, 18] that the coexistence of AF and  $d\text{SC}$  generates the  $\pi$ -triplet order, which should therefore be considered on an equal footing with AF and  $d\text{SC}$ . Our preliminary calculations, however, show that its effects are not strong enough to modify the present conclusions.

Now we discuss implications for experiments, assuming  $t'/t = -1/6$  and  $t''/t = 0$  for LSCO, and  $t'/t = -1/6$  and  $t''/t = 1/5$  for YBCO.[10–12] Compared with the actual phase diagrams, the AF order is overstabilized in Figs. 1(a) and 2(a). The obtained value of  $\delta_{\text{N}}$ , therefore, may be regarded as a rough measure of hole density below which there is a possibility that the AF order is stabilized and coexists with  $d\text{SC}$  especially for LSCO. In this sense, the “1/8 anomalies” are interesting.

The 1/8 anomalies are various anomalies observed around hole density 1/8 in the typical high- $T_c$  cuprates.[19–23] One of anomalies is the possible bulk coexistence of AF and  $d\text{SC}$ . This possibility, however, is reported only in LSCO[9], not in YBCO and Bi2212. In fact,  $\mu\text{SR}$  data show no precession of the muon spin in YBCO[21] and Bi2212[22] even if Zn impurity is introduced. This material dependence will be understood

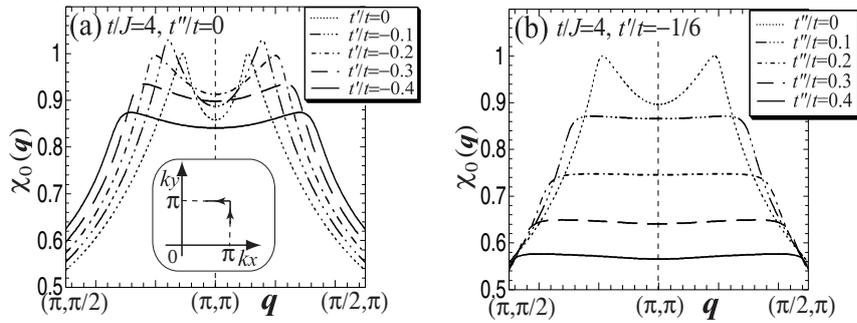


FIG. 5:  $\chi_0(\mathbf{q})$  in the  $d$ RVB state for several choices of  $t'$  (a) and  $t''$  (b) at  $\delta = 0.15$  and  $T = 0.01J$ . The scanned path is shown in the inset of (a).  $\chi_0(\mathbf{q})$  is scaled so that the peak height for  $t'' = 0$  (a) and  $t'' = 0$  (b) is unity.

by the present effect of  $t''$ , since a moderate value of  $t''/t$  ( $\sim 0.2 - 0.3$ ) is expected in both YBCO and Bi2212, and not in LSCO.[10–12]

The search for actual systems, which show a similar phase diagram to Fig. 2(a), is challenging. Such candidates may include Hg- and Tl-based cuprates.

In conclusion, we have studied the possible bulk coexistence of AF and  $d$ SC in the slave-boson scheme of the 2D  $t$ - $J$  model. We have found that  $t''$  has a significant effect on the suppression of the coexistence. This effect will be rather general and appear as noticeable material depen-

dence of the possible bulk coexistence of AF and  $d$ SC.

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