I. INTRODUCTION

Two-dimensional magnetic recording (TDMR) schemes have been one of important element technologies of the recent or future magnetic recording systems and expected to extend the storage density of the magnetic recording systems towards 10 terabits per square inch (Tb/in²). In any TDMR schemes, it is desirable to apply a two-dimensional (2D) signal processing schemes to two-dimensional (2D) data sequences recorded on the magnetic recording medium. But, if we use 2D maximum likelihood (ML) sequence detectors to decode 2D data sequences optimally, there is a difficult computational complexity problem related to NP-hard for ML decoding complexity [1]. Therefore, if we design any 2D modulation code or error-correction code (ECC), we cannot help restricting code rate, encoding or decoding complexity from the viewpoint of signal processing. In this research, it shows the new signal processing system for TDMR. In this proposed system, the design method of 2D modulation codes are based on channel polarization techniques for polar encoding [2]. As a result, these designed 2D modulation codes are equivalent to specific systematic polar codes [3]. Especially, using the fast transform methods in signal processing, it is known that complexity of polar encoding/decoding for the block size \( n \) is \( O(n \log n) \). This characteristic of complexity is preferable for a TDMR scheme because a 2D modulation codeword is decoded by a realistic calculation.

II. TWO-DIMENSIONAL MODULATION CODES USING SYSTEMATIC POLAR CODES

It is known that polar codes are linear block codes which are introduced by Arıkan for effective channel coding [2]. In polar encoding, the polar transform is to apply the transform matrix \( G_2 = \mathcal{G}_2^\otimes n \), the \( n \)-th Kronecker power of the matrix \( G_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \), to the block of \( 2^n \) bits. The polar encoder chooses a set of \( \mathcal{E} \) rows of the matrix \( G_2 \) to a form of \( \mathcal{E} \times 2^n \) matrix which is used as the generator matrix in the encoding procedure. In this encoding process, the information bits which correspond to this set is called “frozen bits”. The way of choosing this set is dependent on the channel and uses a phenomenon called “channel polarization”. Channel polarization is an operation which produces \( 2^n \) channels \( \{ u_i \}_{i=1}^{2^n} \) from \( 2^n \) independent copies of a symmetric binary discrete memoryless channel such that the new parallel channels are polarized in the sense that their mutual information is either close to 0 (completely noisy channels) or close to 1 (perfectly noiseless channels). Every frozen bits are set to dummy symbols “0”. That is, these frozen bits are considered to be parity symbols in a codeword. In this research, the 2D modulation code is defined as follows. In polar encoding, \( Y_{2^n} = G_{2^n} \times U_{2^n} \), where \( Y_{2^n}, U_{2^n} \) are a codeword and a binary information sequence with length of \( 2^n \) bits, respectively. Here, it satisfies that \( G_{2^n} \times G_{2^n} = I_{2^n} \), where \( I_{2^n} \) is the \( 2^n \times 2^n \) identity matrix. That means that systematic encoding with bit-reversal is assumed using a cascade of two non-systematic polar encoder circuits [3]. If a dummy symbol is inserted periodically into a codeword, these dummy symbols give a \( (0, k_x, k_y; N) \) constraint, where a \( (0, k_x) \)-RLL constraint is defined for the down-track direction of each track and a \( k_y \) constraint is defined for the cross-track direction of every \( N \) tracks. In decoding, a codeword without dummy symbols is decoded by a successive cancellation decoder which decodes the bits \( Y_{2^n} \) in order. If \( D_{2^n} = G_{2^n} \times Y_{2^n} \), it is able to obtain estimated information bits.

Hidetoshi Saito
E-mail: h-saito@cc.kogakuin.ac.jp
tel: +81-3-3340-2831
fax: +81-3-3348-3486
III. PROPOSED TDMR SCHEME USING BIT PATTERNAED MEDIA

Fig. 1 shows the block diagram of the TDMR scheme. In this TDMR scheme with bit-patterned media (BPM), it uses the 2D generalized partial response (GPR) equalization system and one-dimensional (1D) channel level log-likelihood ratio (LLR) [3] detector. In Fig.1, \(2N + 1\)-track recording is assumed for the TDMR scheme. For the readback TDMR channel, the readback signal of BPM is represented by the 2D Gaussian pulse response given by [4] and the normalized peak of the pulse amplitude is \(A_p\). The noise sequence is additive white Gaussian noise (AWGN) with zero mean and variance \(\sigma^2\).

IV. ERROR RATE PERFORMANCE OF TDMR SCHEMES

Fig. 2 shows the block error rate (BER) performances of the coded GPR systems. In Fig.2, the solid line shows the BER performance of the proposed coding scheme with the \((0,3,0,3)\) run-length limited (RLL) constraint using four-track recording. The dashed line shows the performance of the conventional binary low-density parity-check (LDPC) coding scheme [5] with the \((0,9)\) RLL constraint using single-track recording. These coding scheme have the effective transmission rate \(\eta_s = 2.6, 0.92\), respectively. The recording condition corresponds to the areal density of 4.0 \(\text{Tb/in}^2\) given by [4]. In this simulation, the SNR is defined as \(\text{SNR} = 20 \log_{10} A_p/\sigma^2\) [dB]. As can be seen Fig. 2, the proposed coding scheme using four tracks outperforms that of the conventional coding scheme using a single track by about 5.0 dB of SNR gains at a BER of \(10^{-5}\).

V. CONCLUSIONS

In this research, new TDMR schemes based on channel polarization are proposed. These proposed schemes have 2D modulation codes which codewords are generated by systematic polar encoding method. Concatenated coding schemes between the 2D modulation code and non-binary LDPC code for four-track recording have the superior performance compared with the conventional 1D high rate binary LDPC coding scheme.

REFERENCES