Creep phenomena of current driven vortices

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Outline

- Introduction
- Details of dynamics simulation
- Simulation results & scaling analysis
- Discussions
- Summary

Introduction

- Physical realizations

  domain walls of ferromagnetic and ferroelectric materials, fluid invasion into porous media, twin boundary, dislocation, CDW, Wigner crystal, Abrikosov vortex lattice, ……

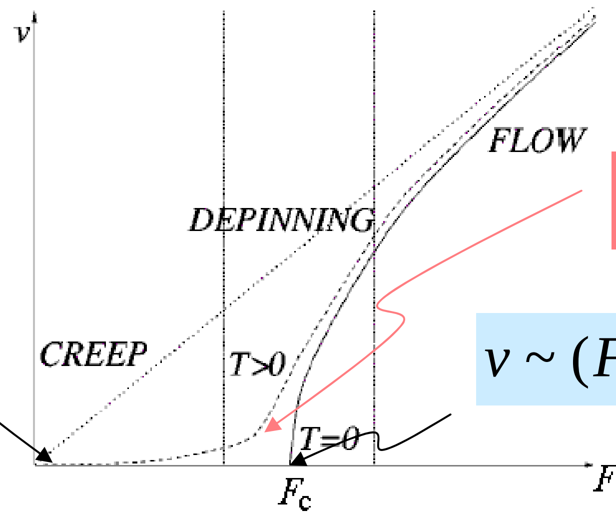
- Key ingredients

  ◇ elastic force: flat structure ◇ random pinning force: deformation

  ➡ nonlinear energy landscape; glassy response

- Typical v-F characteristics

$$v \sim e^{-U_c'(F_c/F)^{\mu}/T}$$



$$v \sim e^{-U_c(1-F/F_c)/T}$$

$$v \sim (F/F_c-1)^{\beta}$$

Theoretical works: review

- D. Fisher (1985): depin transition ~ a critical phenomenon
- Anderson-Kim theory (1964): Arrhenius law $v \sim e^{-(U_0 - Fx)/T}$
- Ioffe and Vinokur (1987), Nattermann (1987), Fiegelmann, Geshkenbein, Larkin and Vinokur (1989):

  highly nonlinear response

  - Larkin and Ovchinnikov (1979): collective pinning
- T. Nattermann (1990): general scaling theory
- Chauve, Giamarchi and Le Doussal (2000): FRG theory
  - M. Muller, D. A. Gorokhov, and G. Blatter (2001)

Dynamics simulation of vortex lines

- overdamped dynamics: $\eta \vec{v}_i = \vec{F}$ $\eta=\phi_0 H_{c2}/\rho_n$: Bardeen-Stephen

$$\eta \vec{v}_i = \sum_{i,j;j\neq i} F_{ij}^{VV}(\rho_{ij}, z_{ij}) + \sum_{i,p} F_{ip}^{VP}(\rho_{ip}) + \vec{F}_L + \vec{F}_{th}$$

$$< F_{th}^{\alpha}(t) F_{th}^{\beta}(t') >= 2\eta k_B T \delta_{\alpha\beta} \delta(t-t')$$

- Details of simulation:
  - $N_v=180$: number of flux lines

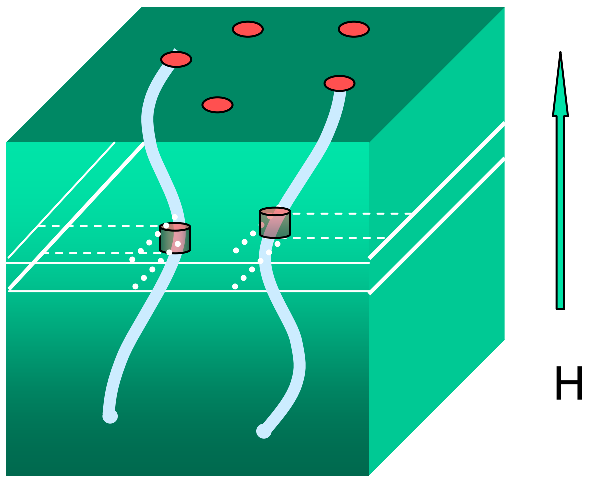
    $N_p=900$: number of pins per layer

    $N_z=20$: number of CuO layers

    $L_{xy}=30\lambda_{ab}$: lateral system size

    B=300 G taking λ_{ab}=2000A

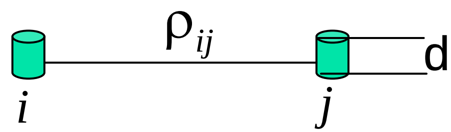
  - 2^{nd} order Runge-Kutta: $\Delta t=0.003$



Forces

Olive et al. PRL, 91 (2003) 037005 and refs. therein

- Intraplane interaction

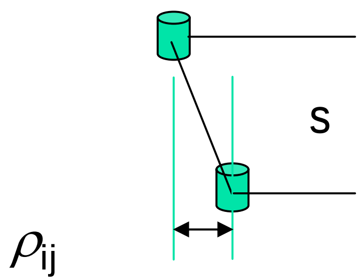


$$U^{VV}(\rho_{ij}) = \varepsilon_0 d K_0(\rho_{ij}/\lambda_{ab})$$

$$F^{VV}(\rho_{ij}) = \varepsilon_0 d/\lambda_{ab} K_1(\rho_{ij}/\lambda_{ab})\hat{\rho}_{ij}$$

  - cut off: $6\lambda_{ab}$

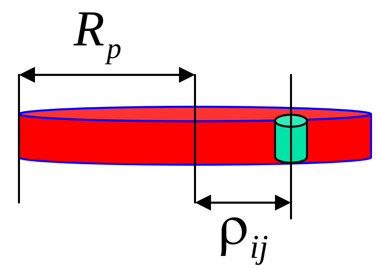
- Interplane interaction



$$U^{VV}(\rho_{ij}) = \frac{s\varepsilon_0}{\pi}\left(1+\ln\frac{\lambda_{ab}}{s}\right)\left[\left(\frac{\rho_{ij}}{2r_g}\right)^2 - 1\right] \qquad \rho_{z,z\pm s} < 2r_g$$

$$\frac{s\varepsilon_0}{\pi}\left(1+\ln\frac{\lambda_{ab}}{s}\right)\left[\left(\frac{\rho_{ij}}{r_g}\right) - 2\right] \qquad \rho_{z,z\pm s} > 2r_g$$

- Pinning force



$$U^{VP}(\rho_{ij}) = -\alpha A_p \exp[-(\rho_{ij}/R_p)^2]$$

$$A_p = \frac{\varepsilon_0 d}{4}\ln[1+(R_p{}^2/2\xi_{ab}{}^2)]$$

$$F^{VP}(\rho_{ij}) = -\frac{2\alpha A_p \vec{\rho}_{ij}}{R_p{}^2}\exp[-(\rho_{ij}/R_p)^2]$$

α : pinning strength

Units and parameters

- units:

energy: $\varepsilon_0 d = \dfrac{\Phi_0^2 d}{8\pi^2\lambda_{ab}^2}$ length: λ_{ab}

force: $F_0 = \varepsilon_0 d/\lambda_{ab}$ time: $\eta\lambda_{ab}/F_0$

temperature: $T_0 = \varepsilon_0 d/k_B$

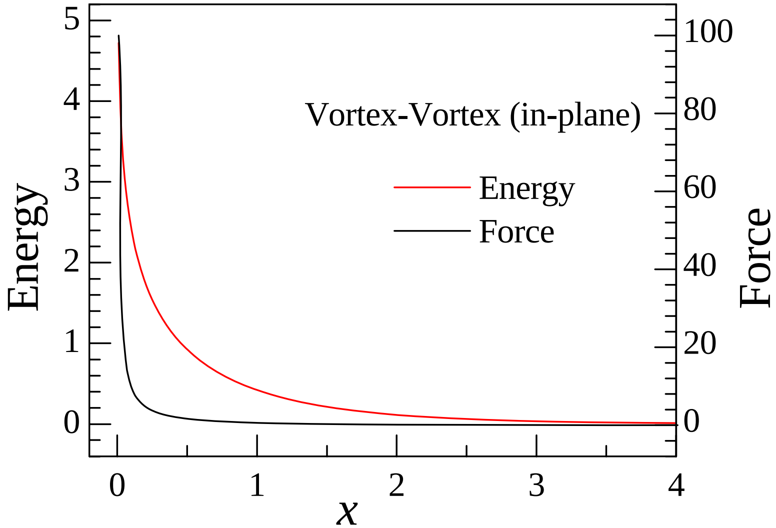
- material parameters:

$s = 8.33 \times 10^{-3}\,\lambda_{ab}$, $d = 2.83 \times 10^{-3}\,\lambda_{ab}$

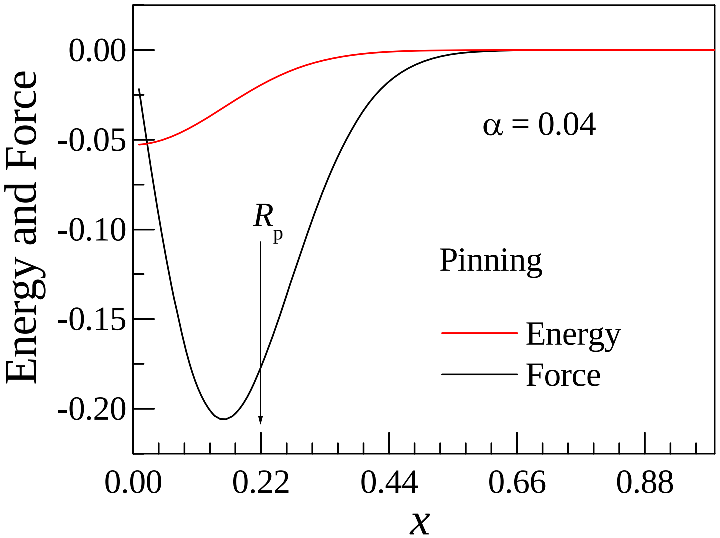
$r_g = \gamma\xi_{ab} = 1.1\,\lambda_{ab}$, $\kappa=90$, $\gamma=100$

$R_p = 0.22\,\lambda_{ab}$

- pancake repulsion



- pinning force

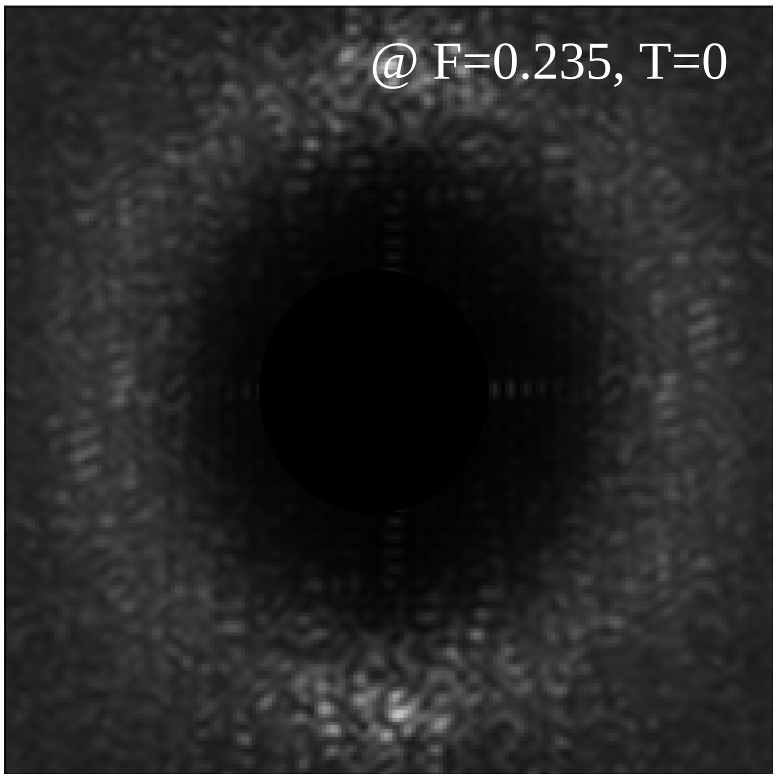


Vortex glass case

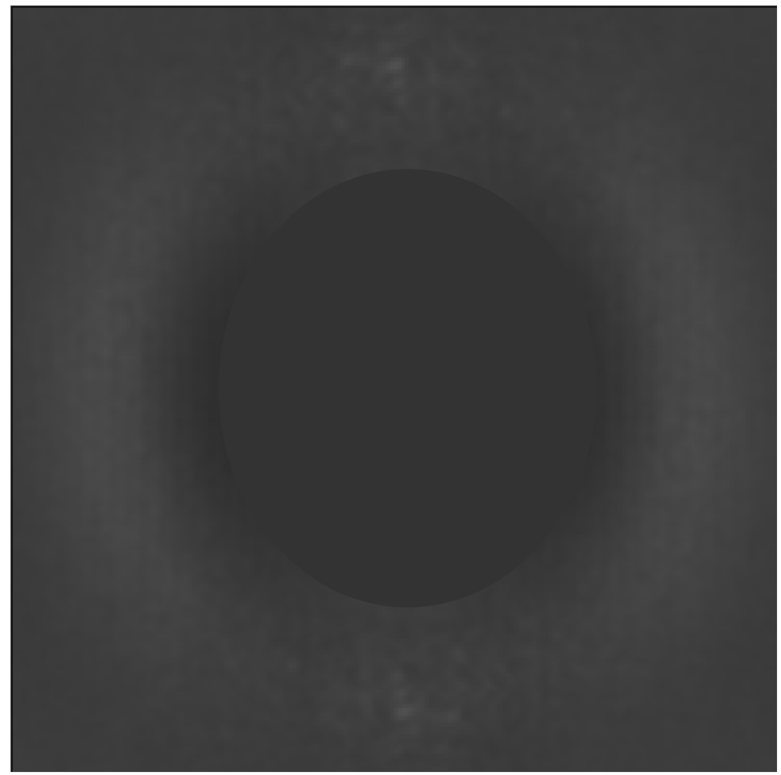
α=0.2

- Structure factor: $$S(\vec{k}) = \frac{1}{N_v^{\,2}} \left| \sum_{j=1}^{N_v} v_x(j) \exp(i\vec{k} \cdot \vec{r}_j) \right|^2$$

vortex position

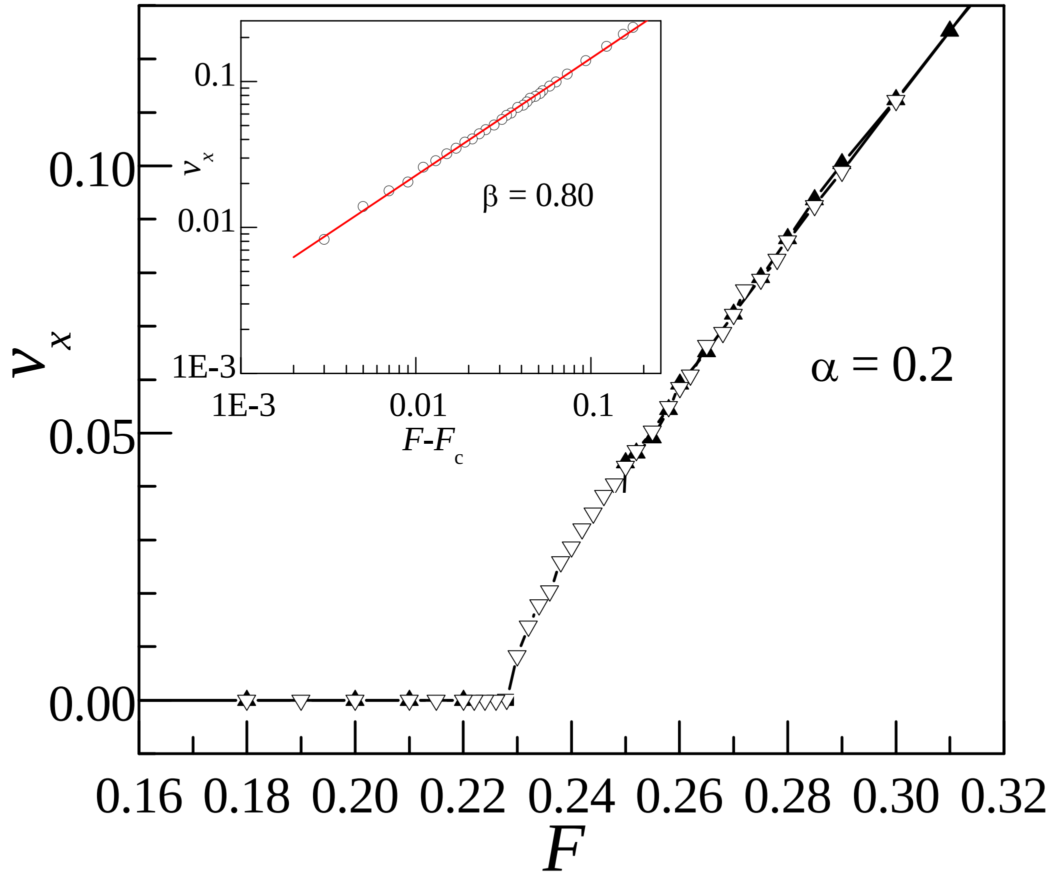


vortex velocity



Depining transition: T=0

- Pinning strength: α=0.2



$$v \sim (F/F_c - 1)^{\beta}$$

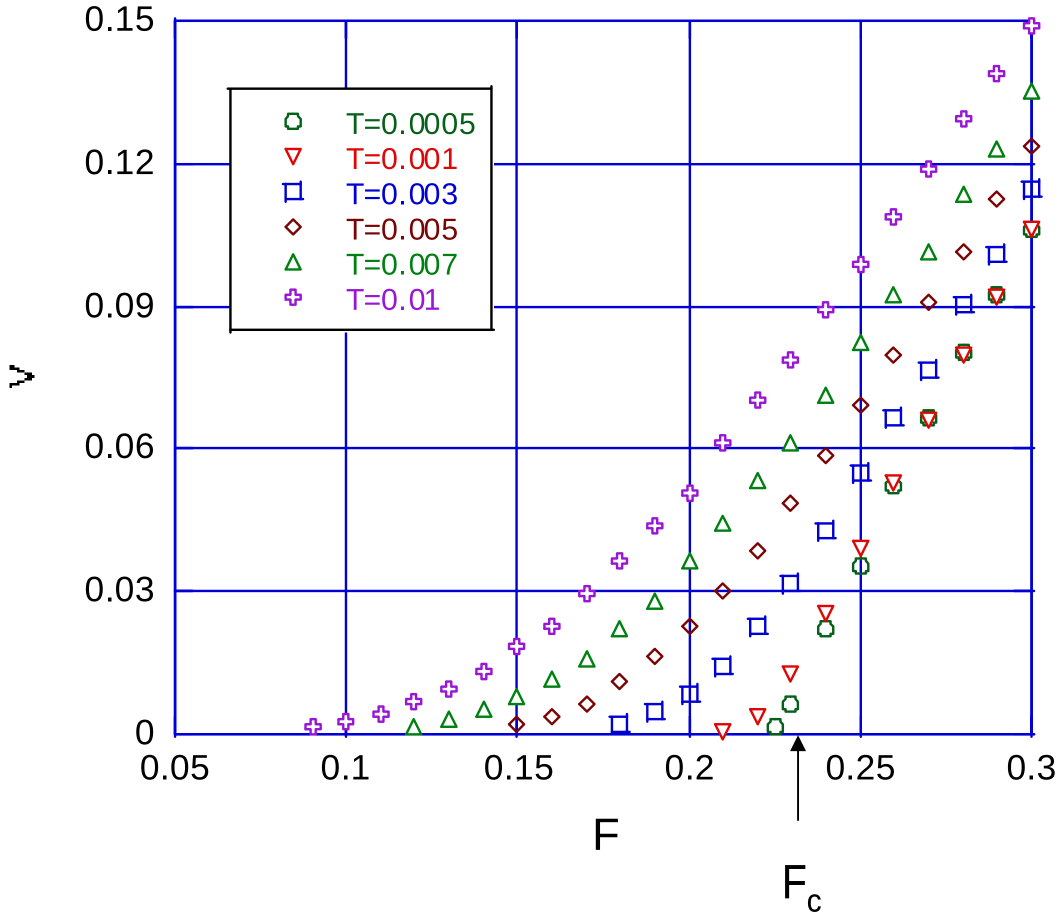
$F_c \sim 0.23$

$\beta \approx 0.8$

ε-expansion

$\beta = 1 - \varepsilon/9$

v-F characteristics: T>0



- Temperature rounds the sharp T=0 depinning transition.
- For T>0, extremely small velocity, no well-defined critical force

Scaling ansatz

- For force around the zero-temperature depinning force F_c, the dynamics can be described by the following scaling law:

$$v(T,F) = T^{1/\delta} S\left(T^{-1/\beta\delta} f\right) \qquad f = \frac{1/F_c - 1/F}{1/F_c} = 1 - \frac{F_c}{F}$$

with the scaling function $S(x) \to x^{\beta}$, as $x \to +\infty$

$$v(T \to 0, F) \sim f^{\beta} \sim \left(F/F_c - 1\right)^{\beta}$$ for $F>F_c$ & $T=0$

- velocity correlation becomes long-ranged as $F \to F_c$ — Avalanches

- Analogy with para-ferro magnetic "transition" in magnetic field

v	⇔	m
T	⇔	H
$f=1-F_c(T=0)/F$	⇔	$t=1-T/T_c(H=0)$

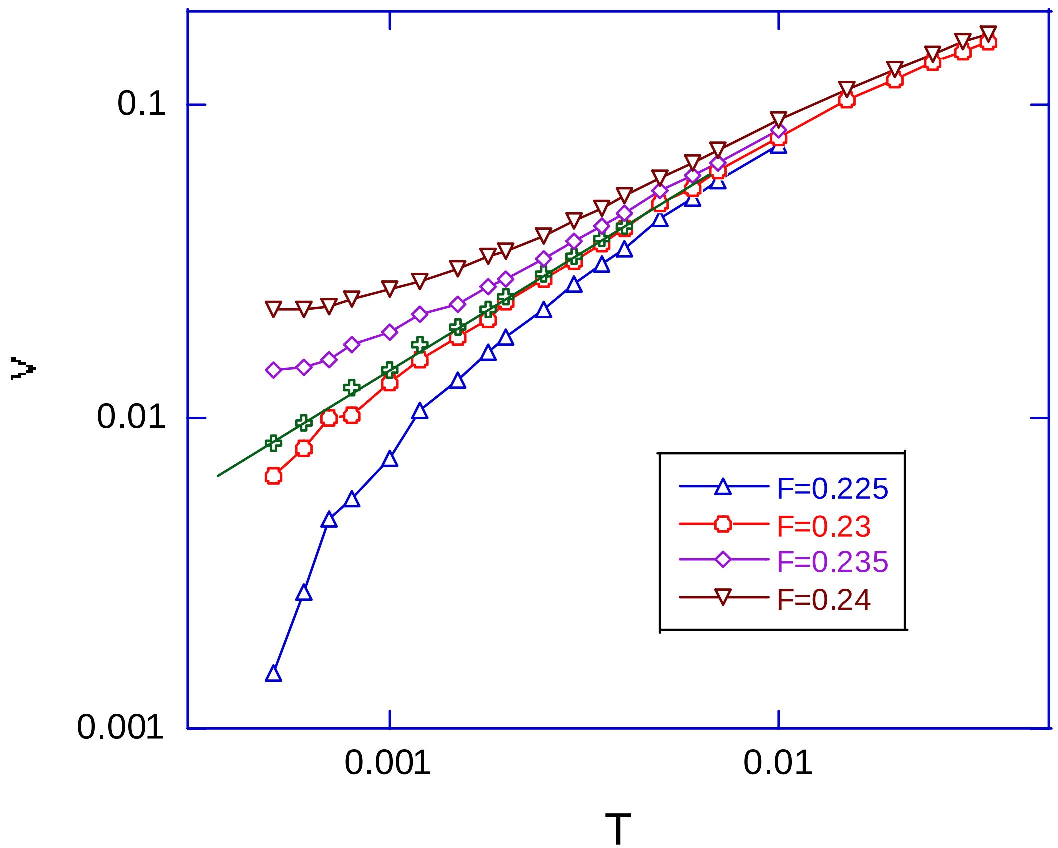
m=M(H,T)

Equation of State

Scaling analysis

- Scaling function: $v(T,F) = T^{1/\delta} S\left(T^{-1/\beta\delta} f\right)$ $\qquad f = 1 - \frac{F_c}{F}$

$v \propto T^{1/\delta}$ @ F=F_c

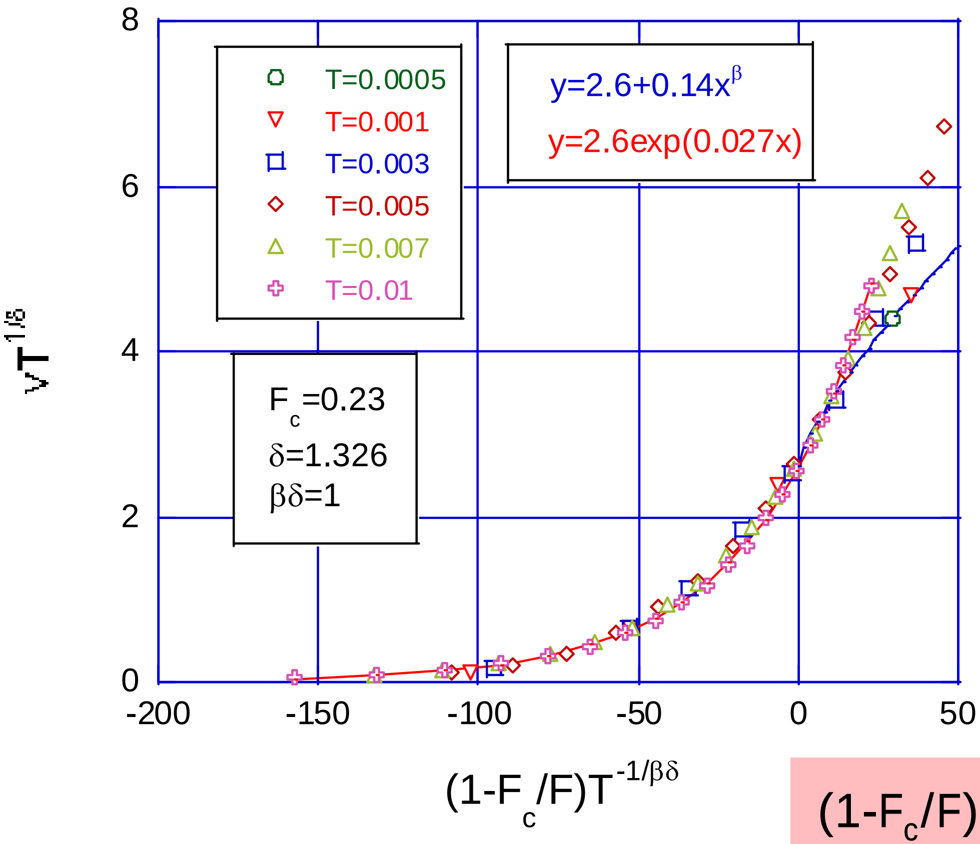


- Critical force and exponent

$$1/\delta = 0.754 \pm 0.010$$
$$F_c = 0.2315 \pm 0.0013$$

Scaling plot

- Scaling function: $v(T,F)T^{-1/\delta} = S\left(T^{-1/\beta\delta} f\right)$



$(1-F_c/F)\ /T$

- Depin transition: $F>F_c$

$$v(T\to 0, F) = 0.14(1-F_c/F)^{\beta}$$

$$\beta = 1/\delta = 0.754 \pm 0.010$$

- Creep motion: $F_c/2<F<F_c$

$$v = v_0 T^{1/\delta} \exp\left[-\frac{U_c}{T}\left(\frac{F_c}{F}-1\right)\right]$$

Arrhenius law !

$T \leq U_c/3$

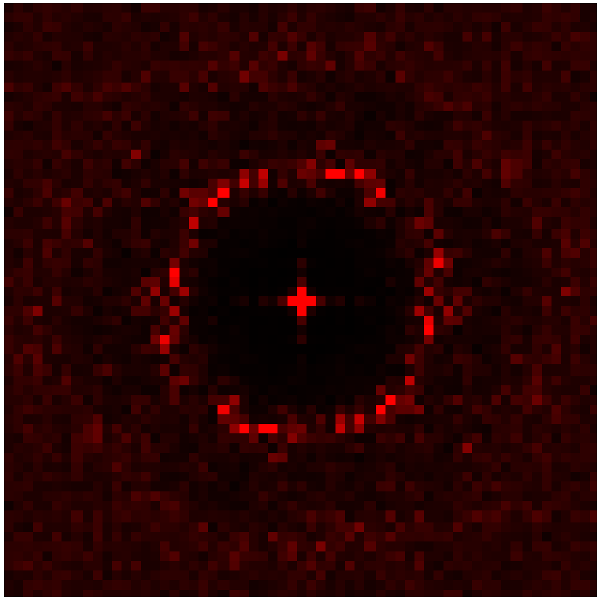
$v_0=2.6,\ \ U_c=0.027$

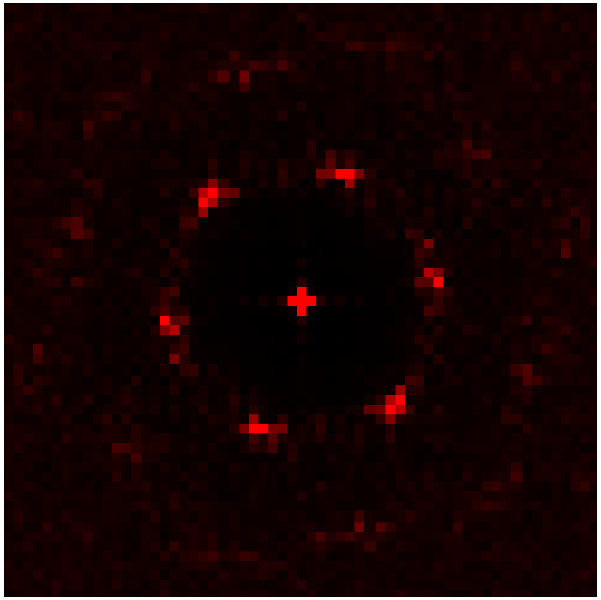
Bragg glass case

α=0.05

- Structure factor at equilibrium

T = 0.08 T = 0.075

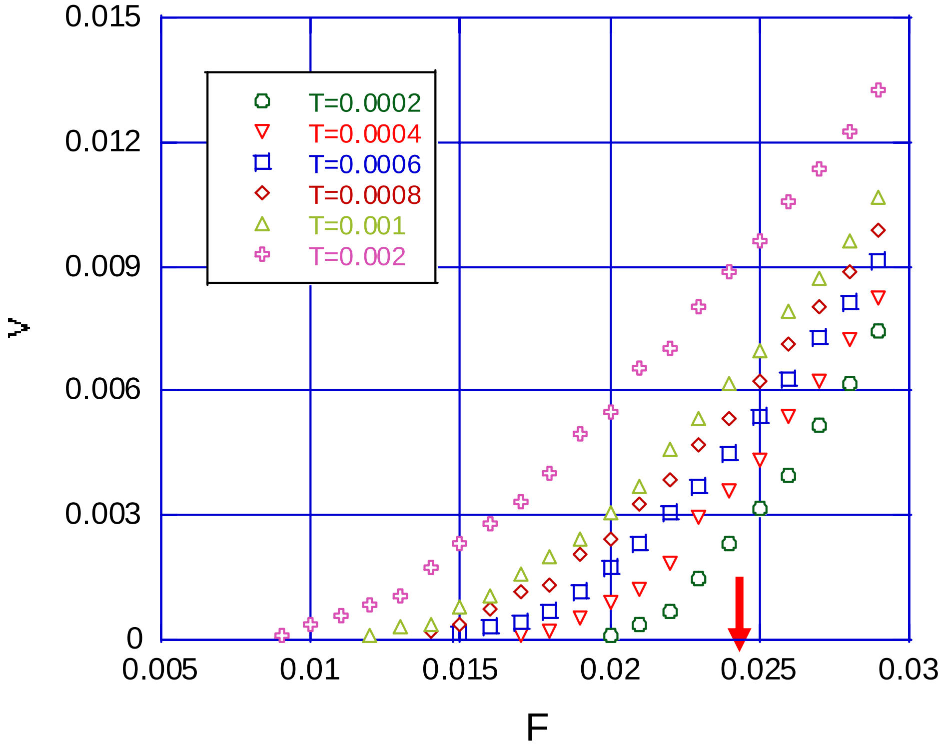


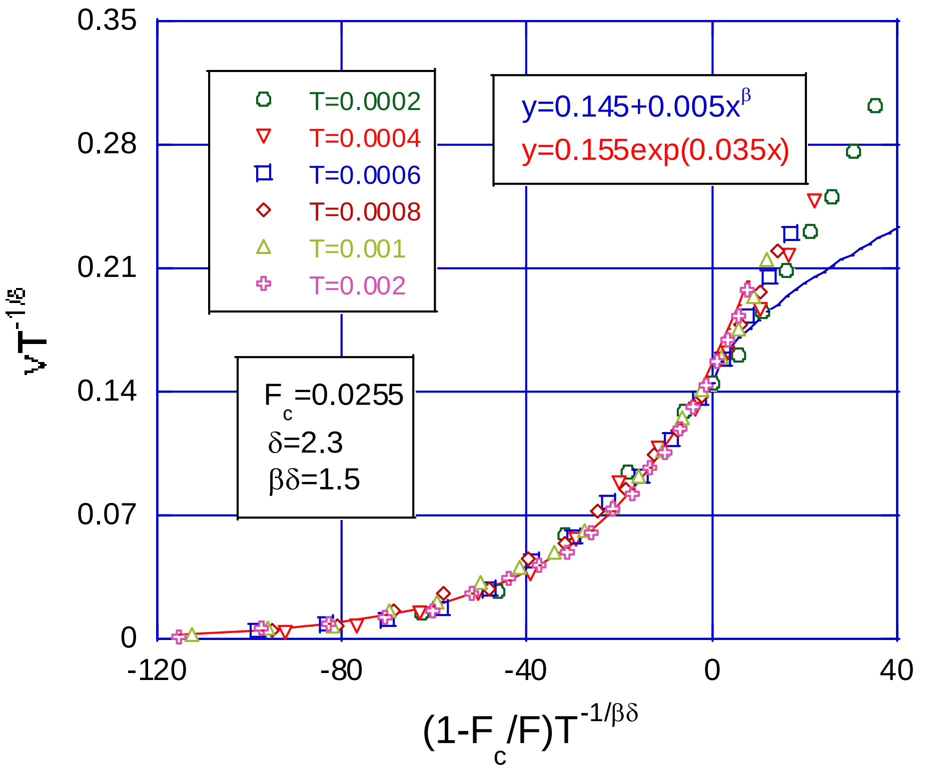


T_m=0.077

Bragg glass case

- dynamics $v(T,F)=T^{1/\delta}S\left(T^{-1/\beta\delta}f\right)$

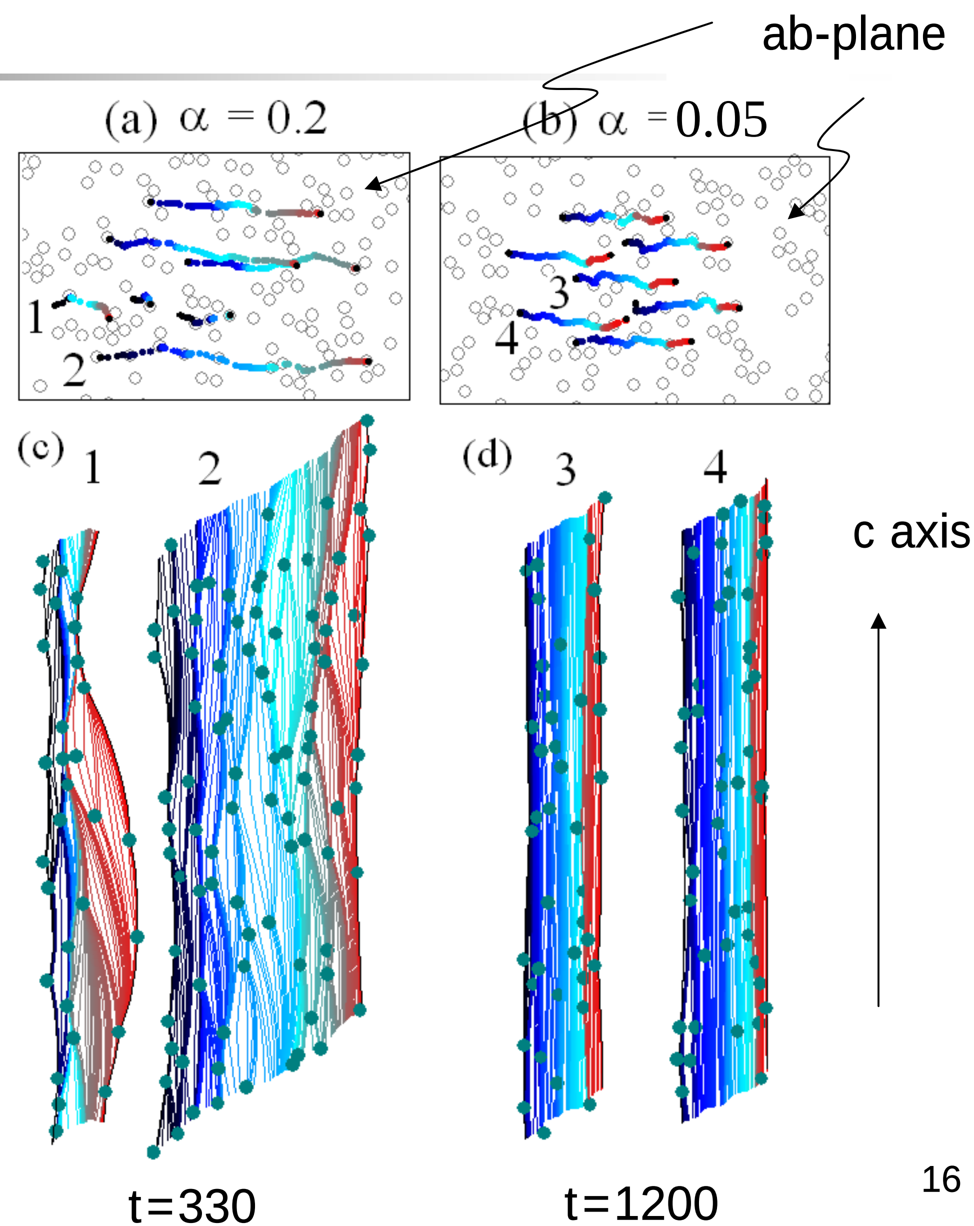




- creep law: $v\sim T^{1/\delta}\exp\left((1-F_c/F)(U_c/T)^{2/3}\right)$ non-Arrhenius type
- exponents: $\beta=0.65;\quad \delta=2.3$ Cf. ε-expansion: β=1-ε/6

“Microscopics” of vortex motion

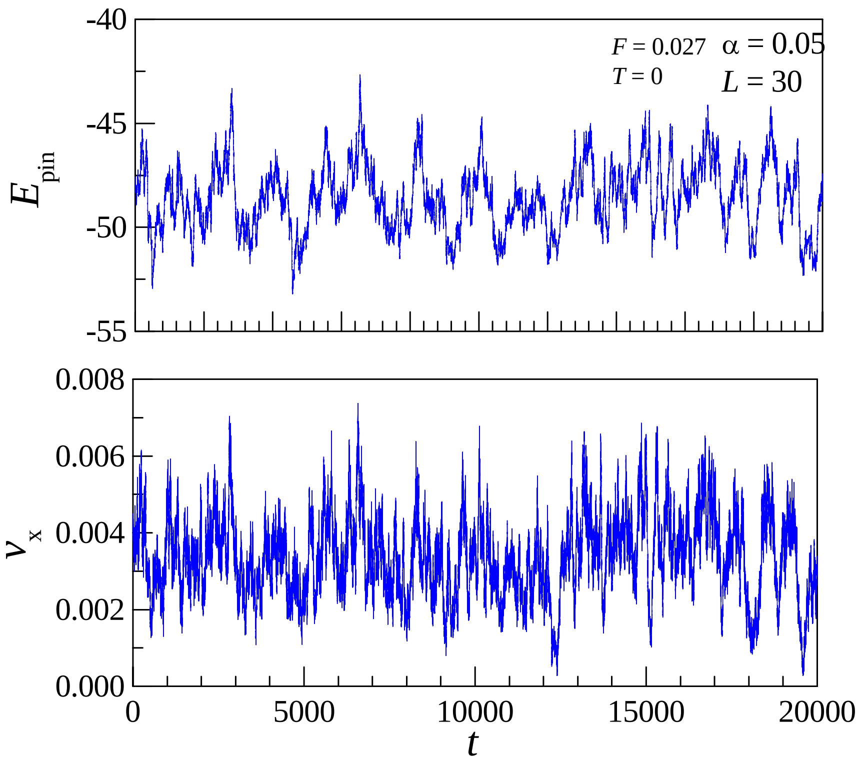
- vortex configuration
  - (a) & (c): VG case
    - random motions in scale both larger and smaller than a_0 ➔ dislocations
  - (b) & (d): BG case
    - random motions in scale smaller than a_0; long-range correlated in scale larger than a_0
- local, intermittent motions sliding & pinned, ...

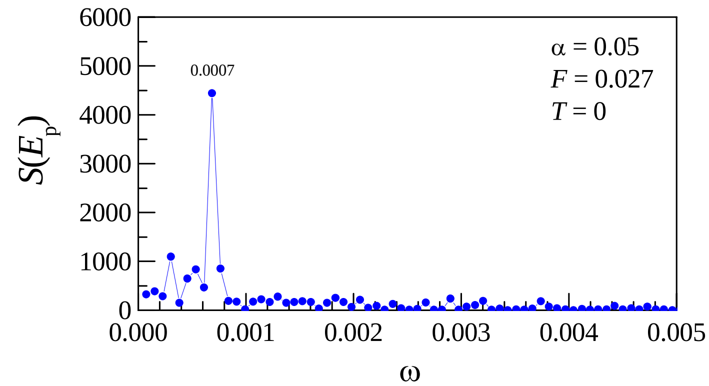


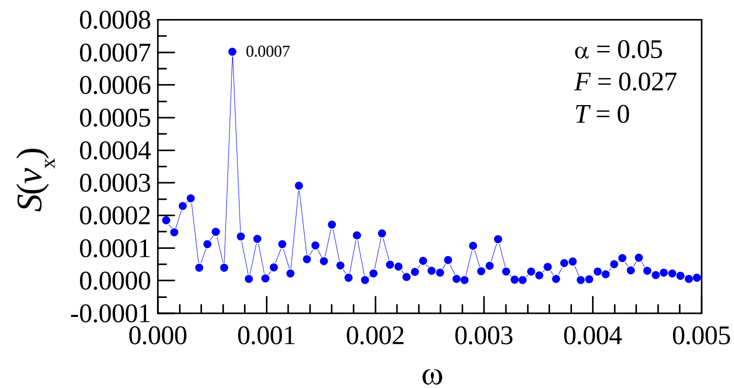
Washboard noises

- temporal correlation

$$S_P(\omega) = \frac{1}{T}\left|\int_0^T dt P(t) e^{i2\pi\omega t}\right|^2$$







$\langle v \rangle = 0.00351$

$S/\langle v \rangle^2 = 56.82$

"Microscopics" of vortex motion

- energy barrier

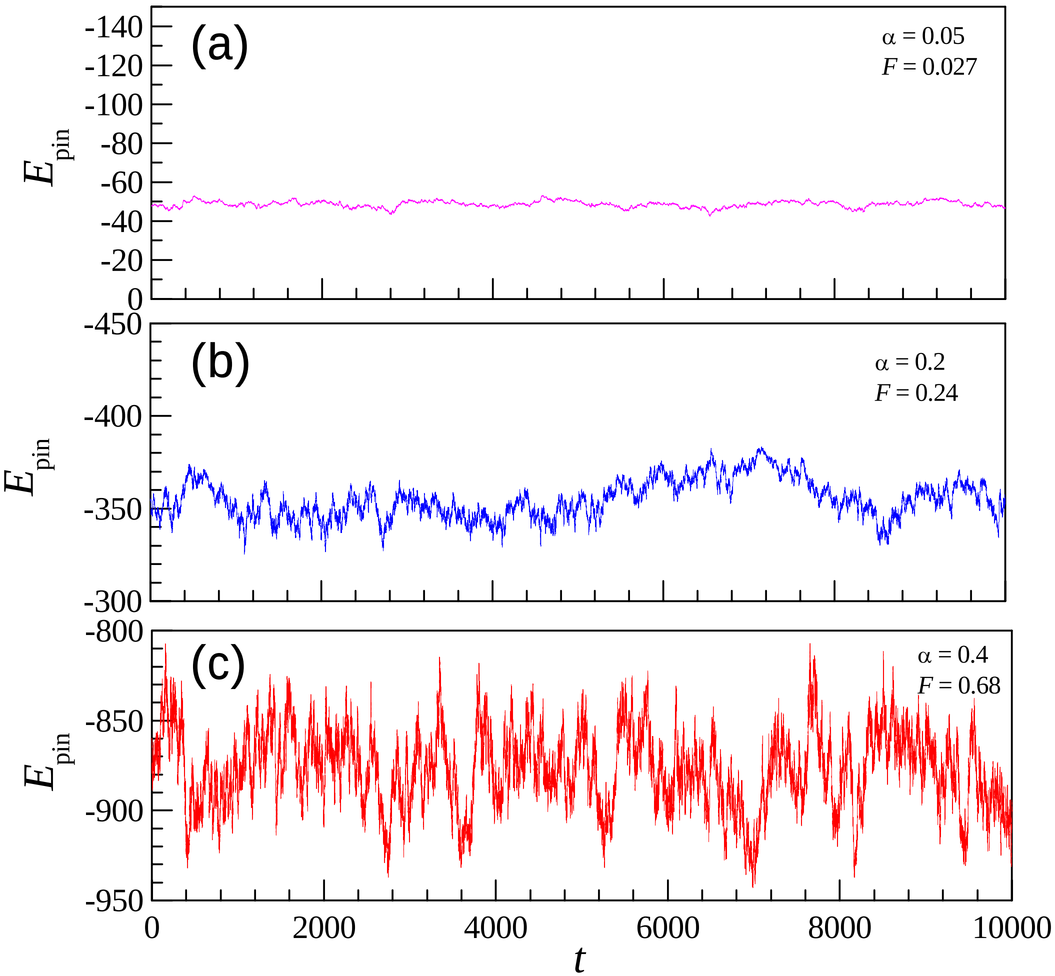
(a): Bragg-glass case

(b): α=0.2
(c): α=0.4
vortex-glass case

Energy barrier increases with pin strength α

- small energy barrier for BG

⇔ local patches move in an intermittent way

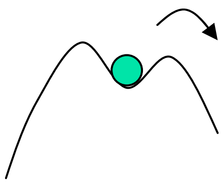


Discussion

- MF nucleation process:

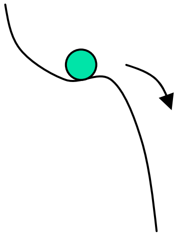
  (A) near coexistence curve; (B) near spinodal

- Case A: $E = Fx + x^2 - x^4$



  - barrier height: $\Delta E \sim (\text{const} - F)$
  - scaling relation: $(\text{const} - F)/T$

- Case B: $E = Fx - x^3$



  - barrier height: $\Delta E \sim (\text{const} - F)^{3/2}$
  - scaling relation: $(\text{const} - F)/T^{2/3}$

- Examples of nonlinear creep
  - flux fluctuations in SQUID
  - friction between STM tip & substrate
  - Mott hopping conductance

Summary

- Scaling property around depining force F_c

$$v(T,F) = T^{1/\delta} S\left(T^{-1/\beta\delta}(1 - F_c/F)\right)$$

Equation of state

- ◇ depining at T=0: $v \propto (F/F_c - 1)^{\beta}$
- ◇ creep at $F_c/2 < F \leqslant F_c$ at $T \ll T_m$: $v \propto T^{1/\delta} \exp\left(-(F_c/F - 1)(U_c/T)^{1/\beta\delta}\right)$
- ◇ strong pinning (⇔VG): $\beta\delta=1$

  Arrhenius law with linearly suppressed energy barrier

  - ○ M. Muller, D. A. Gorokhov, and G. Blatter (2001)
- ◇ weak pinning (⇔ BG): $\beta\delta=1.5$

  stretched exponential → non-Arrhenius type

Need a theory !

Thank you!