Nano Superconductivity as a Novel Source of Terahertz Electromagnetic Wave

S.-Z. Lin and X. Hu
Nano-System Theoretical Physics Group
WPI Center for Materials Nanoarchitectonics
National Institute for Materials Science

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Outline

- Introduction
  - THz electromagnetic wave
  - Josephson effect
  - recent experimental breakthrough

- theoretical model

- new dynamic state in Josephson junctions: $\pi$ kink state

- Summary
Spectrum of electromagnetic wave
Introduction: Terahertz EM wave

- Important applications range from DNA diagnosis to security check
  - vibration modes of proteins and DNA molecules in the THz range.

- THz imaging

Introduction


The diagram illustrates the THz gap, a region in the spectrum where current technology is limited. It shows the relationship between frequency (f [THz]) and power (in mW) for various devices and applications:

- **Microwaves** (low frequency, high power)
- **Far infrared light** (high frequency, low power)
- **Quantum cascade laser**
- **Impatt**
- **Gunn**
- **Photomixing**
- **III-V laser**
- **Optics**

The THz gap is marked by a triangle, indicating a deficiency in technology to bridge this region efficiently.
Superconductivity: a macro quantum state

Wave function of junction system:

\[ H\psi_n = E_n\psi_n + J_0(\psi_{n+1} + \psi_{n-1}) \]

 Analogy with hoping particle:

position \( x \)  \quad momentum \( k \)  \quad number \( n \)  \quad phase \( \gamma \)

Bloch theorem: hoping on uniform 1D lattice

\[ \psi_k = \sum_v \psi_v e^{ikv} \quad E(k) = E + 2J_0 \cos k \]

Superconductivity: broken gauge symmetry
\( \Leftrightarrow \) fixed phase of w.f.
\( \Leftrightarrow \) coherent condensate of many states with different number of Cooper pairs
Josephson effect

- dc Josephson relation:
  \[ I = I_c \sin \gamma \]

- ac Josephson relation:
  \[ V = \frac{\hbar}{2e} \frac{d\gamma}{dt} \]

- THz oscillation
  \[ f = 0.483 \left( \frac{\text{THz}}{\text{mV}} \right) \times V[\text{mV}] \]

- Group velocity
  \[ \frac{\hbar d\langle n \rangle}{dt} = \frac{\partial E(k)}{\partial k} \sim \sin k \]

- Force equation: voltage V
  \[ E_{v+1} - E_v = 2eV \]
  \[ \frac{d\langle \hbar k \rangle}{dt} = 2eV \]

- Continuous radiation possible in principle

The positions x, momentum k, and phase \( \gamma \) are connected in a cycle.
BSCCO as Intrinsic Josephson Junctions

- Discovery of BSCCO-2212
  \( \gamma : 80 \sim 1000 \)

- Evidences of Josephson effect
  R. Kleiner et al. PRL 68, 2394 (1992)

\[ \frac{s + D}{\lambda_{ab}} \ll 1 \]

weak screening \( \leftrightarrow \) superconductivity in the nano scale

Courtesy of K. Kadowaki

- EM radiation at 0.6THz
- Energy enhancement of 3 orders

Key experimental results:

(I) frequency and voltage obey the ac Josephson relation

(II) frequency of radiation follows the cavity relation: $\lambda_{EM} = 2w$

(III) coherent state along the c-axis $\leftrightarrow$ radiation power $\sim N^2$

(IV) radiation in narrow regime of voltage $\rightarrow$ cavity resonance
Basic equations for multi junctions

- Coupled sine-Gordon equations

\[
\Delta P_l = \left(1 - \zeta \Delta^{(2)}_l \right) \sin P_l + \partial_t^2 P_l + \beta \partial_t P_l - J_{\text{ext}}
\]

- \( \Delta^{(2)}_l Q_l = Q_{l+1} + Q_{l-1} - 2Q_l \)

- ac Josephson relation: \( \partial_t P_l = E_l^z \)

- GL relation for junction: \( \partial_x P_l = \zeta s \left( J_{l+1}^x - J_l^x \right) + B_l^y \)

- Total current: \( J_l^z = \sin P_l + \beta E_l^z + \partial_t E_l^z \)

- Maxwell equation

\( \nabla \times \mathbf{B} = \mathbf{J} \)

- Current conservation

\( \text{div} \mathbf{J} = 0 \)

Sakai et al. (1993); Kleiner et al. (1994) Koyama et al. (1996); Bulaevskii et al. (2006)
Mesa sample as a cavity

Neumann-type boundary condition

\[ \partial_n P = 0 \]

Small mesa thickness: \( L_z/\lambda_{EM} = 1/300 << 1 \)

\( \Rightarrow \) tangential magnetic field is vanishingly small \( \Rightarrow \) establish a cavity

- Neumann-type boundary condition
- Small mesa thickness: \( L_z/\lambda_{EM} = 1/300 << 1 \)
- Tangential magnetic field is vanishingly small
- Establish a cavity

New solution: Kink state

- Form of solution:
  \[ P_i(x, t) = \omega t + A \cos \frac{\pi x}{L} \sin(\omega t + \phi) + f_l P^s(x) \]

- \[ f_l = (-1)^l \]
- \[ f_l = (-1)^{[l/2]} \]

- ac Josephson term
- Josephson plasma term
- coupling term

- Why \( \cos(kx) \)?

  eigen function of Laplace equation satisfying b.c. \( \partial_x P \big|_{edge} = 0 \)

- \( E^z = \partial_t \tilde{P}, \quad B^y = \partial_x \tilde{P} \)
Emergence of dc supercurrent

- Expansion of sine of sine in terms of Bessel function

\[
\exp(iz \sin \omega t) = \sum_{n=-\infty}^{\infty} J_n(z) \exp(in\omega t)
\]

\[
\sin[(\omega t + f_lP^s + z \sin(\omega t + \varphi)] = \sum_{n=-\infty}^{\infty} J_n(z) \sin[(n+1)\omega t + n\varphi + f_lP^s]
\]

- Physics of Shapiro steps: appearance of dc component

- CSG equation for the solution

\[
-(\pi/L)^2 Ag_{10}^m \sin(\omega t + \varphi) + f_l \partial_x^2 P^s =
\]

\[
\beta \omega - J_{ext} - A \omega^2 g_{10}^m \sin(\omega t + \varphi) + A \beta \omega g_{10}^m \cos(\omega t + \varphi)
\]

\[
+ (1 - \varsigma \Delta^{(2)}) \sum_{n=-2,-1,0} J_n \left(Ag_{10}^m\right) \sin[(n+1)\omega t + n\varphi + f_lP^s]
\]

up to fundamental frequency: \( \sin(\omega t) \& \cos(\omega t) \)
Eigen state of operator $\Delta^{(2)}$

- **difference operator** $\Delta^{(2)}$

$$
\Delta^{(2)}Q_l = Q_{l+1} + Q_{l-1} - 2Q_l
$$

- Eigen state of period 2-layer for

$$
f_l = (-1)^l \quad +---+-++-
\Delta^{(2)} \sin(f_lP^s) = -4 \sin(f_lP^s)
$$

- Eigen state of period 4-layer

$$
f_l = (-1)^{[l/2]} \quad +--+-+-+-
\Delta^{(2)} \sin(f_lP^s) = -2 \sin(f_lP^s)
$$

- For the two states the CSG eqs are decoupled

$$
\iff \Delta^{(2)} \cos(f_lP^s) = 0
$$

They are stable, and observed in simulations frequently.
\[ \frac{d^2 P^s}{dx^2} = q \zeta \cos \varphi J_{-1}(Ag_{10}^m) \sin P^s \]  

(1)

boundary condition: \( \partial_x P^s \big|_{\text{edge}} = 0 \)

width of kink: \( \frac{1}{\sqrt{\zeta}} \)

\[ g_{10}^m(x) = \cos \frac{\pi x}{L} \]
Texture of superconductivity phase along c axis

- Phase difference evolves with time linearly in accordance with the ac Josephson relation.
- Static part $P_s^c: \pi$ phase kink
dc supercurrent

- IV characteristics

\[ g_{10}^m(x) = \cos \frac{\pi x}{L} \]

\[ J_{\text{ext}} = \beta \omega - \frac{\sin \phi}{L} \int_0^L J_1(A g_{10}^m) \cos P^s \, dx = \beta \omega \left(1 + \frac{A^2}{4}\right) \] (2)

- the kink \( P^s \) permits pumping large current and energy

- Coupling between dc driving and transverse plasma

  - cf. Koshelev and Bualevskii

The new kink state builds up strong coupling automatically.
IV characteristics: current step at cavity voltage

- Cavity mode: \( c' = c / \sqrt{\varepsilon_c} \)
- \( \omega_{\text{cavity}} = \frac{\pi}{L_x} = 7.854 \leftrightarrow \approx 0.6 \text{THz} \)
- \( V = \phi_0 / 2 \sqrt{\varepsilon_c L_x} = 1.225 \text{[mV]} \)
- ac Josephson relation holds
- negative differential resistance
  - cf. ZFS in single junction

- Small A approximation: take linear terms in (3) & (4)

\[
J_{\text{ext}} = \beta \omega \left[ 1 + \frac{(I_{10}^m)^2}{(\omega^2 - (\pi / L)^2)^2 + (\beta \omega)^2} \right]
\]

A. Koshelev, PRB 78, 174509 (2008)

\[
I_{10}^m = \frac{1}{L_x} \int_0^{L_x} g_{10}^m \cos P^s dx \approx 2 / \pi
\]
Radiation power: simulation

Pyonting vector at the junction edges

\[ P = 2000 \times L_z \times L_y = 6 \text{ mW} \]

Radiated energy: taking the mesa in the experiments

\[ L_x = 80 \mu\text{m} \]
\[ \zeta = 4.44 \times 10^5 \]
\[ \beta = 0.02 \]
\[ \varepsilon_r = 10 \]
\[ |Z| = 1000 \]
\[ \theta = \pi/4 \]

Optimal value upon Z variation
Metaphor

Zigzag Staircase

Wedged Blocks

$+\pi$ kink
$-\pi$ kink
Kink state: (1,1) mode of rectangular mesa

**Solution:**

\[ P_t(x, y, t) = \omega t + A \cos \frac{\pi x}{L_x} \cos \frac{\pi y}{L_y} \sin(\omega t + \varphi) + f_t P^s(x, y) \]

**2D phase kink:**

\[ \left( \partial_x^2 + \partial_y^2 \right) P^s = q \zeta \cos \varphi J_{-1}(A g_{11}^m) \sin P^s \]

\[ g_{11}^m(x, y) = \cos(\pi x/L_x) \cos(\pi y/L_y) \]

\[ \omega_{11} = \sqrt{\left(\pi/L_x\right)^2 + \left(\pi/L_y\right)^2} \]

K. Kadowaki et al. Physica C 468, 634 (2008); and preprints
Cylindrical geometry

\[ P_l(r, t) = \omega t + AJ_1 \left( \frac{v_{11}^c}{a} \rho \right) \cos \phi \sin(\omega t + \varphi) + f_l P^s(r) \]

\[ f_{11}^c = \frac{1.8412}{a} \]

\[ E^z(r, t) = \partial \tilde{P}/\partial t \]

\[ \mathbf{B}(r, t) = -\nabla \times \left( \tilde{P}(r, t) \mathbf{z} \right) \]
We found a new dynamic state in Josephson junctions

- $\pi$ kink in superconductivity phase

This is a device converting large dc energy into THz radiation.

- the basic principle is the Josephson effect.
- feeding grass to get milk, pumping water to get oil
- efficiency $<10\%$, dissipations inevitable, no free lunch!

Our theory explains all the important features of experiments so far.

The theoretical picture is based on large-scale computer simulations

- intuition very limited for highly nonlinear sine-Gordon systems