Nano Superconductivity as a Novel Source of Terahertz Electromagnetic Wave

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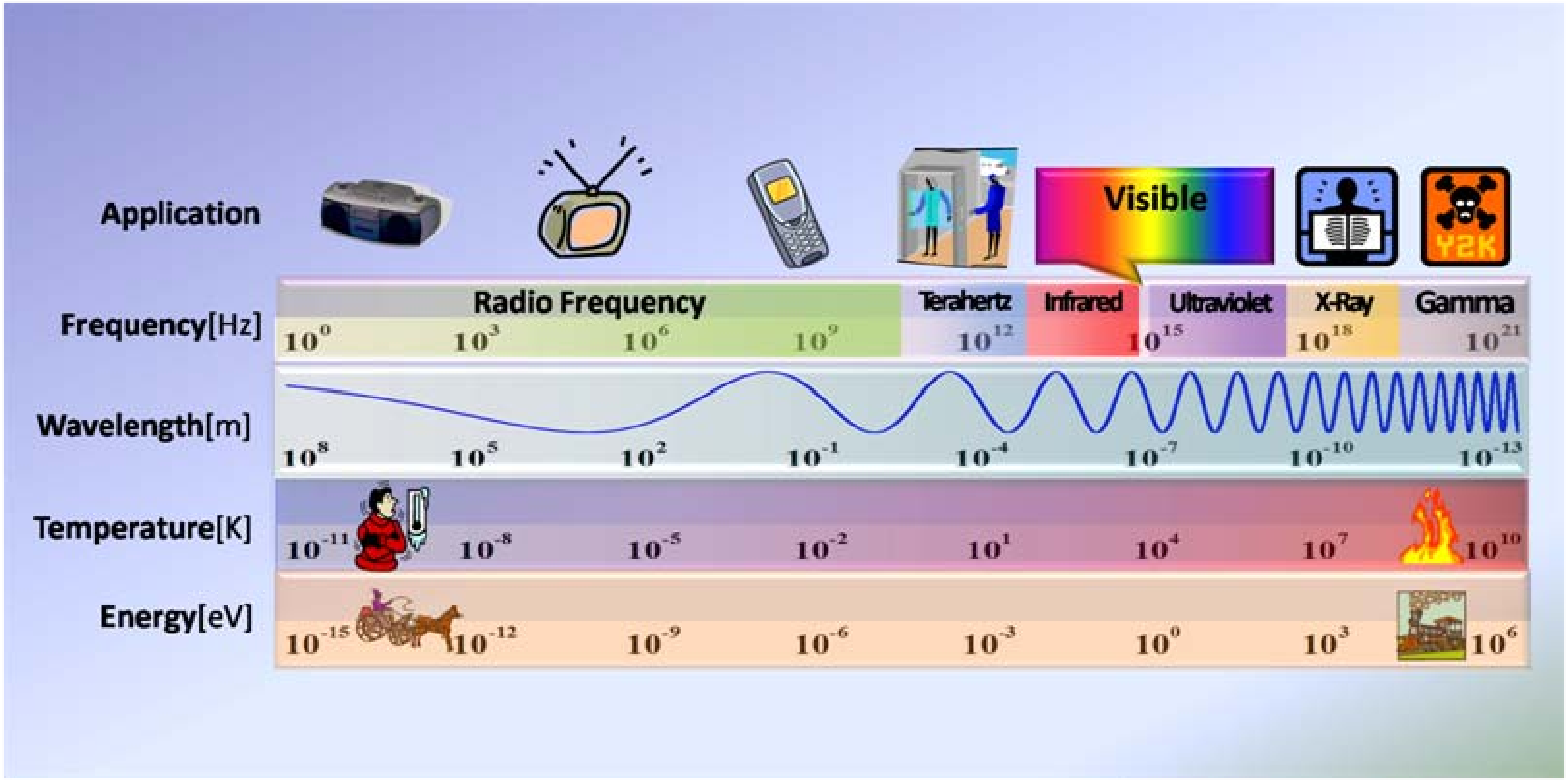
S.Z. Lin and XH, PRL **100**, 247006 (2008)
XH and S.-Z. Lin, PRB **78**, 134510 (2008)

Outline

- Introduction
  - THz electromagnetic wave
  - Josephson effect
  - recent experimental breakthrough
- theoretical model
- new dynamic state in Josephson junctions: π kink state
- Summary

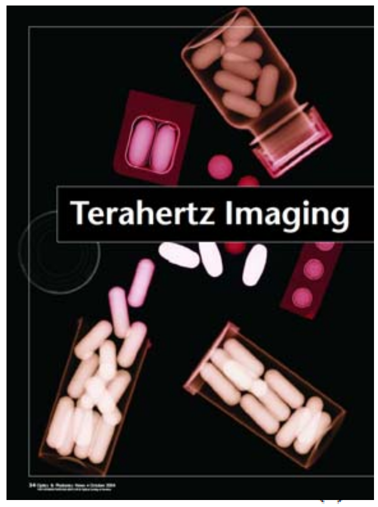
Introduction

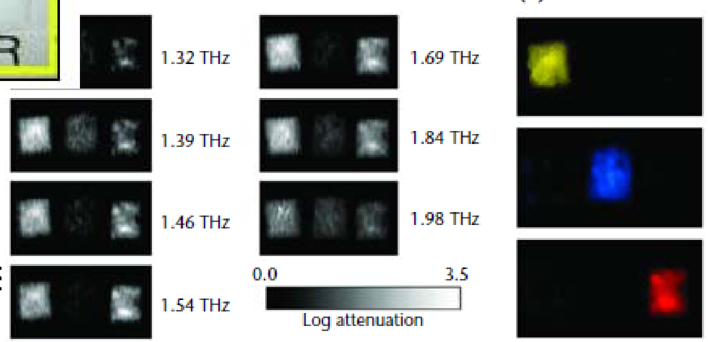
- Spectrum of electromagnetic wave

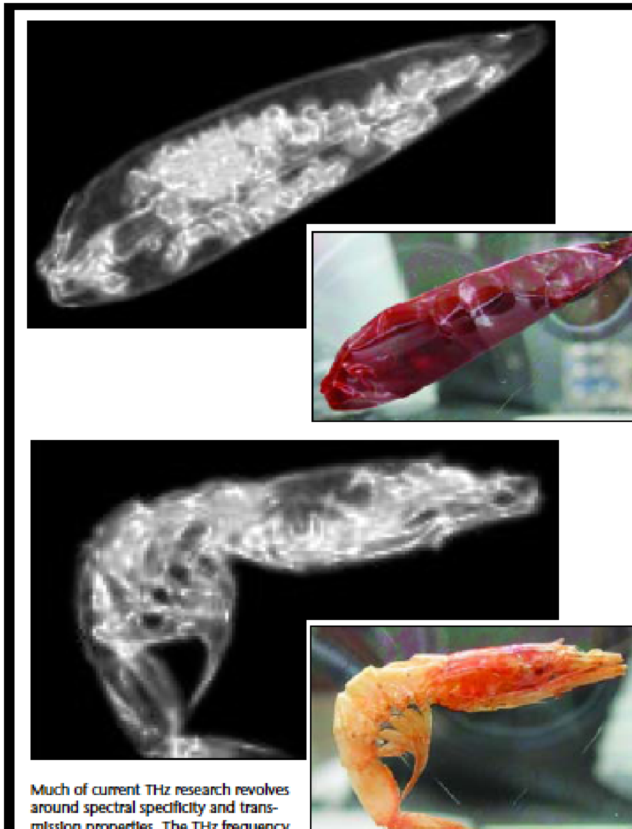


Introduction: Terahertz EM wave

- Important applications range from DNA diagnosis to security check
  - vibration modes of proteins and DNA molecules in the THz range.
- THz imaging





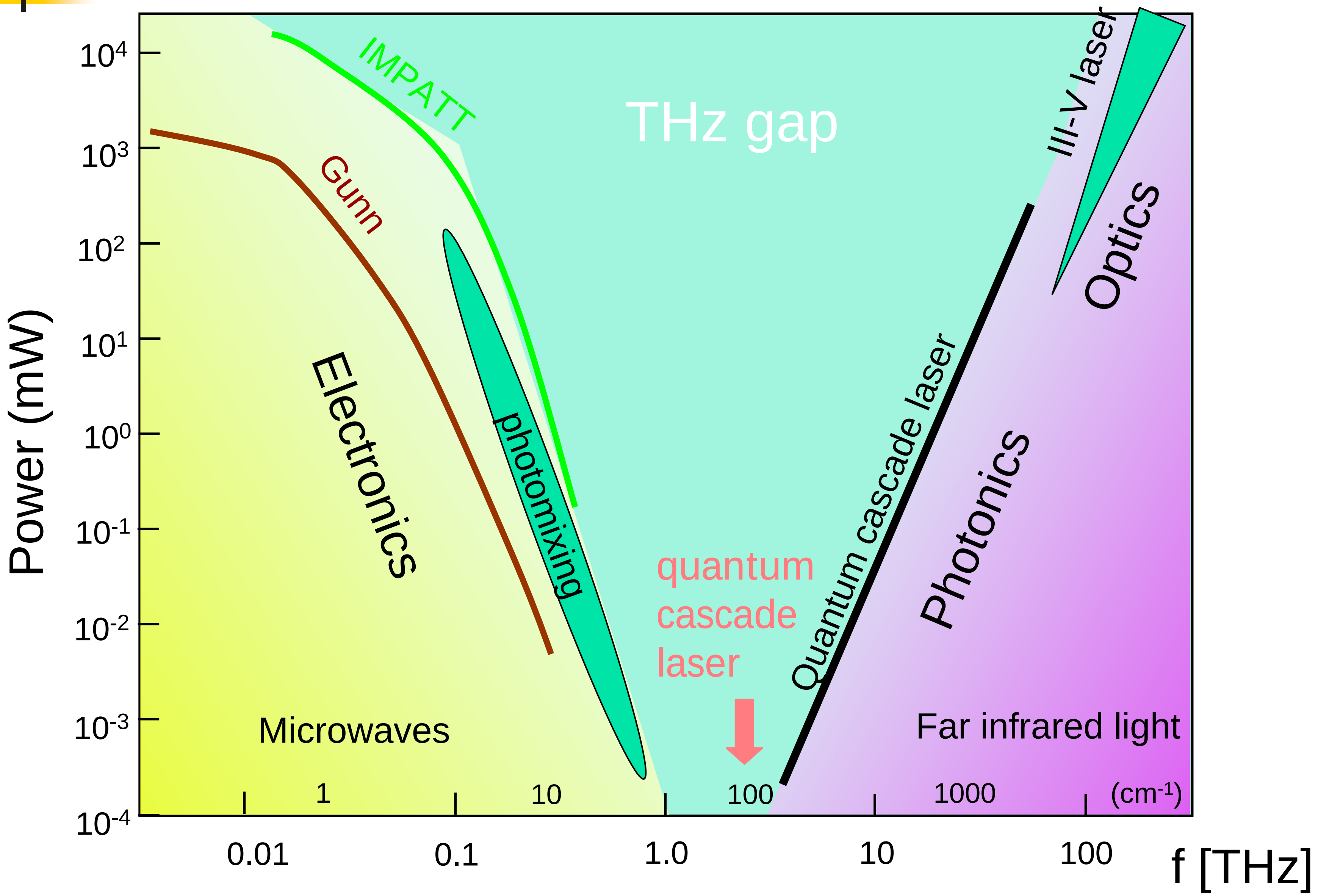


Much of current THz research revolves around spectral specificity and transmission properties. The THz frequency is more selective than X-rays and is therefore more sensitive to the nature of the materials it passes through. Here, a red pepper and a prawn are presented alongside their THz images. [Yuichi Oshima, RIKEN.]

Ref. K. Kawase,
Optics and Photonics News
Oct. 2004, p.38

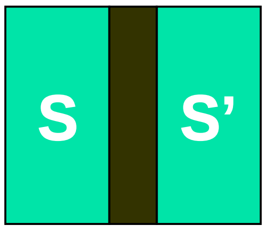
Introduction

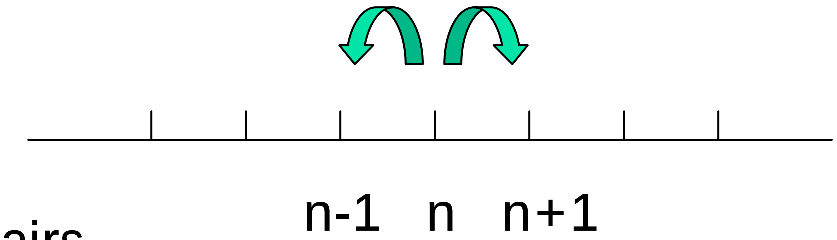
M. Tonouchi, Nature Photonics, vol. 1, 97 (2007)



Superconductivity: a macro quantum state

- Wave function of junction system: $H\psi_n = E_n\psi_n + J_0\left(\psi_{n+1} + \psi_{n-1}\right)$





N-n n # of Cooper pairs

- Analogy with hoping particle:

quantum conjugate quantities

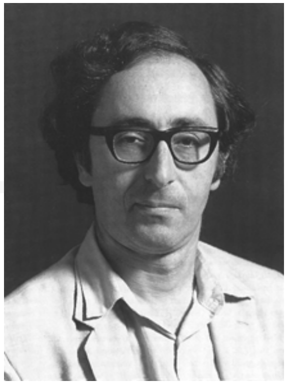
position x	number n
momentum k	phase γ

- Bloch theorem: hoping on uniform 1D lattice

$$\psi_k = \sum_\nu \psi_\nu e^{ik\nu} \qquad E(k) = E + 2J_0 \cos k$$

Superconductivity:
broken gauge symmetry
⇔fixed phase of w.f.
⇔coherent condensate of many states with different number of Cooper pairs

Josephson effect



- dc Josephson relation:

$$I = I_c \sin\gamma$$

- ac Josephson relation:

$$V = \frac{\hbar}{2e}\frac{d\gamma}{dt}$$

- THz oscillation

$$f = 0.483\left[\frac{\mathrm{THz}}{\mathrm{mV}}\right]\times V[\mathrm{mV}]$$

- Group velocity

$$\frac{\hbar d\langle n\rangle}{dt} = \frac{\partial E(k)}{\partial k} \sim \sin k$$

- Force equation: voltage V

$$E_{\nu+1} - E_\nu = 2eV$$

$$\frac{d\langle \hbar k\rangle}{dt} = 2eV$$

position x	⟷	number n
momentum k		phase γ

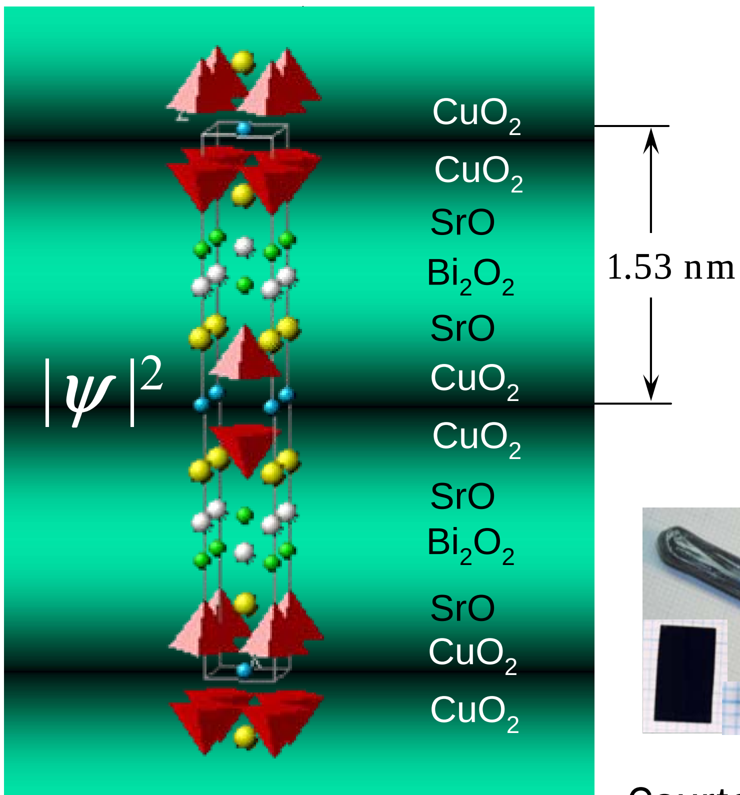
- Continuous radiation possible in principle

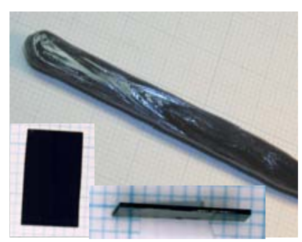
BSCCO as Intrinsic Josephson Junctions

- Discovery of BSCCO-2212

H. Maeda et al. Jpn. J. Appl. Phys. 27, L209 (1988)

γ: 80~1000

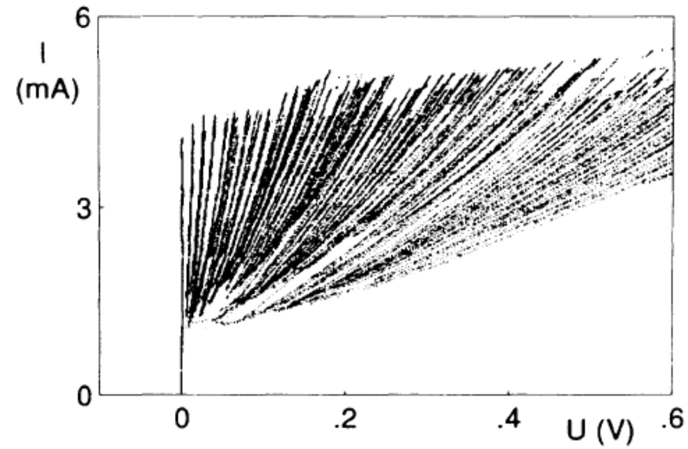




Courtesy of K. Kadowaki

- Evidences of Josephson effect

R. Kleiner et al. PRL 68, 2394 (1992)

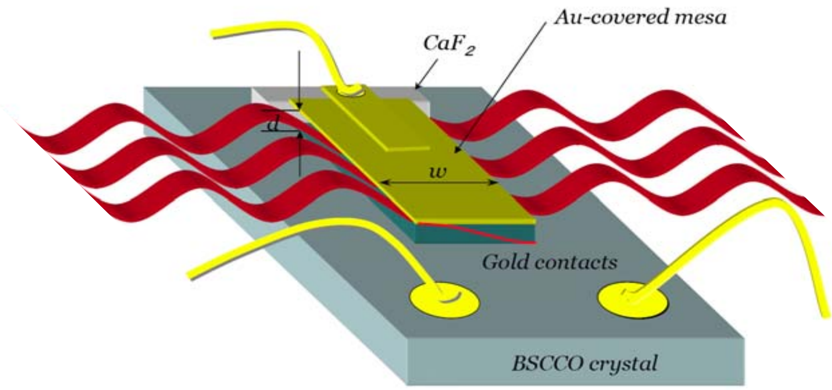


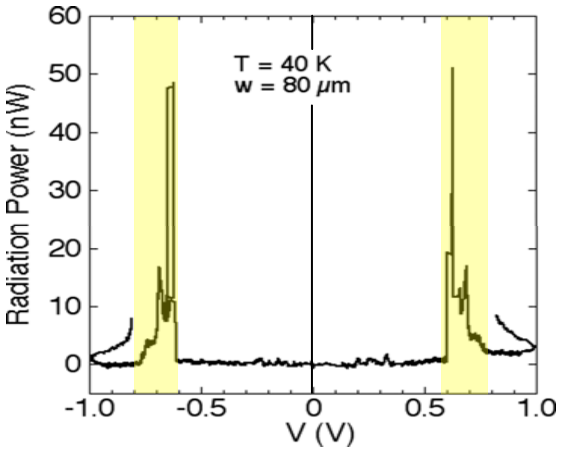
$$\frac{s+D}{\lambda_{ab}} << 1$$

weak screening ← superconductivity in the nano scale

Introduction

- New breakthrough: L. Ozyuzer et al., Science **318**, 1291 (2007)
  - EM radiation at 0.6THz
  - energy enhancement of 3 orders





- Key experimental results:

  (I) frequency and voltage obey the ac Josephson relation

  (II) frequency of radiation follows the cavity relation: $\lambda_{EM} = 2w$

  (III) coherent state along the c-axis ← radiation power~N^2

  (IV) radiation in narrow regime of voltage → cavity resonance

Basic equations for multi junctions

- Coupled sine-Gordon equations P_l: gauge invariant phase difference

$$\Delta P_l = \left(1 - \varsigma\Delta^{(2)}\right)\left(\sin P_l + \partial_t^2 P_l + \beta\partial_t P_l - J_{ext}\right)$$

$$\varsigma = \frac{\lambda_{ab}^2}{(s+D)^2}$$

$$\beta = \frac{4\pi\sigma_c\lambda_c}{c\sqrt{\varepsilon_c}}$$

  - $$\Delta^{(2)}Q_l = Q_{l+1} + Q_{l-1} - 2Q_l$$

- ac Josephson relation: $\partial_t P_l = E_l^z$

- GL relation for junction: $\partial_x P_l = \varsigma s\left(J_{l+1}^x - J_l^x\right) + B_l^y$

- Total current: $J_l^z = \sin P_l + \beta E_l^z + \partial_t E_l^z$

- Maxwell equation

$$\nabla \times \mathbf{B} = \mathbf{J}$$

- Current conservation

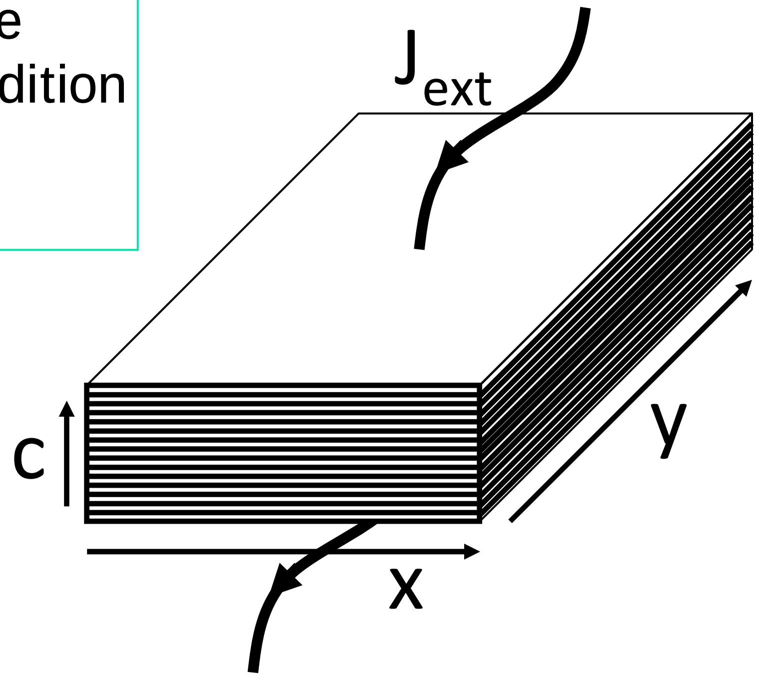
$$\mathrm{div}\mathbf{J} = 0$$

Sakai et al. (1993);
Kleiner et al. (1994)
Koyama et al. (1996);
Bulaevskii et al. (2006)

Mesa sample as a cavity

Neumann-type boundary condition

$$\partial_n P = 0$$



- parameters

L_x=80μm

L_y=300μm

L_z=1μm

$$\varsigma = 4.44\times10^5$$

$$\beta = 0.02$$

$$\varepsilon_c = 10$$

$$\lambda_c = 200\mu m$$

$$\gamma = 500$$

Small mesa thickness: L_z/λ_{EM}=1/300<<1

➔ tangential magnetic field is vanishingly small ➔ establish a cavity

Refs. Bulaevskii & Koshelev: PRL (2007)

New solution: Kink state

- Form of solution:

$$P_l(x,t) = \omega t + A\cos\frac{\pi x}{L}\sin(\omega t + \varphi) + f_l P^s(x)$$

$$f_l = (-1)^l$$

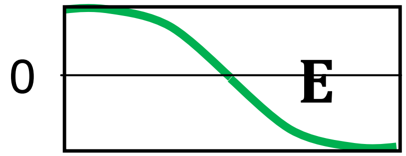
$$f_l = (-1)^{[l/2]}$$

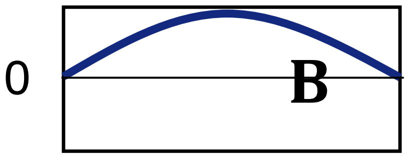
- ac Josephson term
- Josephson plasma term
- coupling term

- Why cos(kx) ?

eigen function of Laplace equation satisfying b.c. $\partial_x P|_{edge} = 0$

- $E^z = \partial_t \tilde{P}, \quad B^y = \partial_x \tilde{P}$





Emergence of dc supercurrent

- Expansion of sine of sine in terms of Bessel function

$$\exp(iz\sin\omega t) = \sum_{n=-\infty}^{\infty} J_n(z)\exp(in\omega t)$$

$$\sin\left[\omega t + f_l P^s + z\sin(\omega t + \varphi)\right] = \sum_{n=-\infty}^{\infty} J_n(z)\sin\left[(n+1)\omega t + n\varphi + f_l P^s\right]$$

  - Physics of Shapiro steps: appearance of dc component

- CSG equation for the solution

$$g_{10}^{m}(x) = \cos\frac{\pi x}{L}$$

$$-(\pi/L)^2 A g_{10}^{m}\sin(\omega t + \varphi) + f_l \partial_x^2 P^s =$$

$$\beta\omega - J_{\text{ext}} - A\omega^2 g_{10}^{m}\sin(\omega t + \varphi) + A\beta\omega g_{10}^{m}\cos(\omega t + \varphi)$$

$$+\left(1 - \varsigma\Delta^{(2)}\right)\sum_{n=-2,-1,0} J_n\left(A g_{10}^{m}\right)\sin\left[(n+1)\omega t + n\varphi + f_l P^s\right]$$

up to fundamental frequency: sin(ωt) & cos(ωt)

Eigen state of operator $\Delta^{(2)}$

- difference operator $\Delta^{(2)}$

$$\Delta^{(2)} Q_l = Q_{l+1} + Q_{l-1} - 2Q_l$$

- Eigen state of period 2-layer for $\Delta^{(2)} \sin\left(f_l P^s\right)$

$$f_l = (-1)^l \qquad \text{+-+-+-+-} \qquad \Delta^{(2)} \sin\left(f_l P^s\right) = -4 \sin\left(f_l P^s\right)$$

q

- Eigen state of period 4-layer

$$f_l = (-1)^{[l/2]} \qquad \text{++--++--} \qquad \Delta^{(2)} \sin\left(f_l P^s\right) = -2 \sin\left(f_l P^s\right)$$

q

- For the two states the CSG eqs are decoupled ⇐ $\Delta^{(2)} \cos\left(f_l P^s\right) = 0$

They are stable, and observed in simulations frequently.

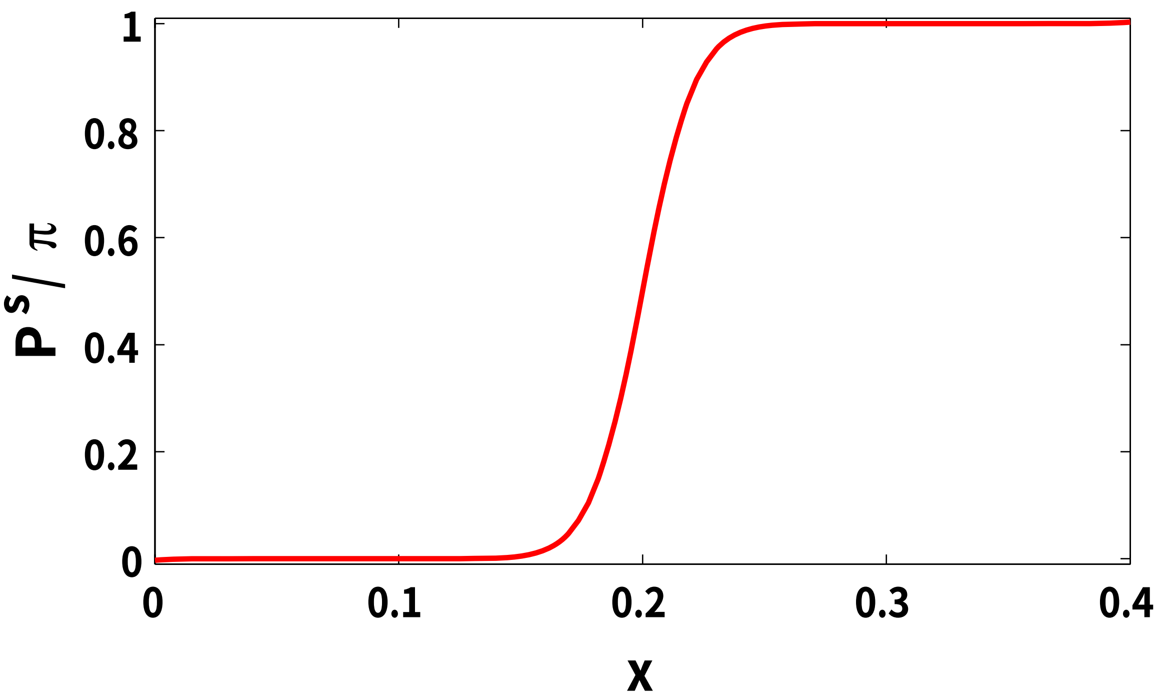
π phase kink

$$g_{10}^{m}(x) = \cos\frac{\pi x}{L}$$

- differential equation for P^s

$$\partial_x^2 P^s = q\varsigma\cos\varphi J_{-1}(A g_{10}^{m})\sin P^s \qquad (1)$$

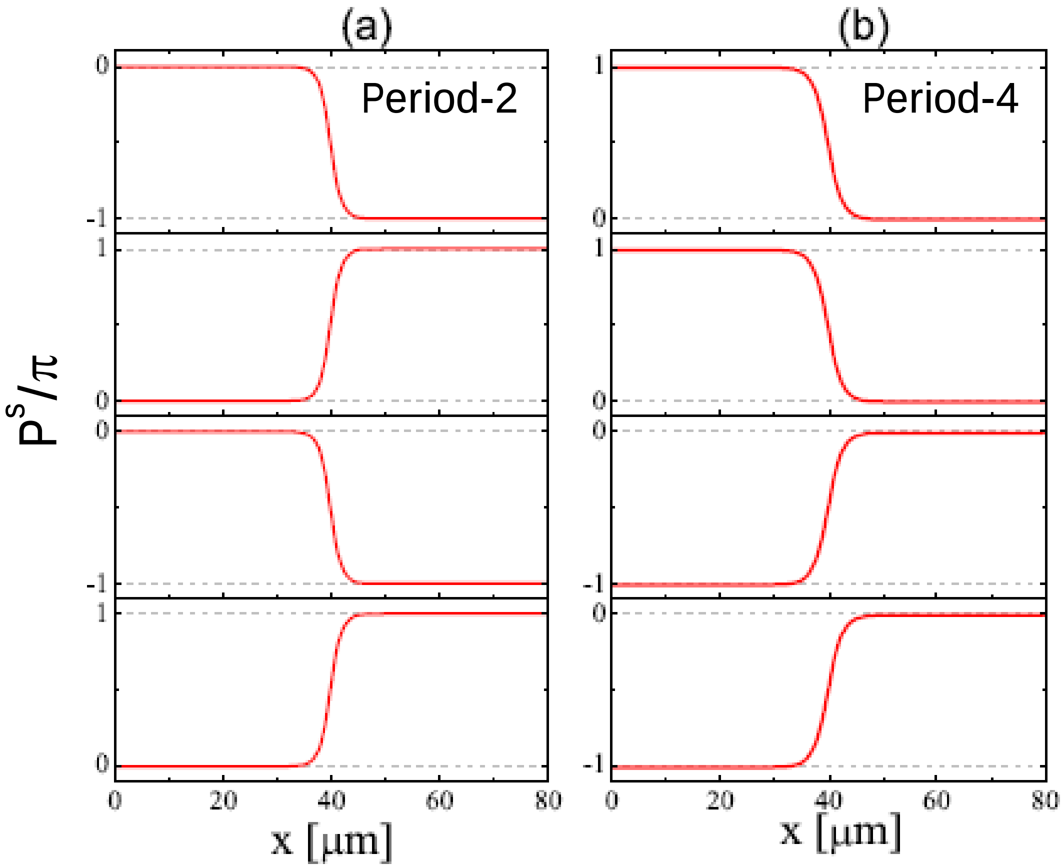
- boundary condition: $\partial_x P^s |_{edge} = 0$
- width of kink: $1/\sqrt{\varsigma}$



π Phase kink

Texture of superconductivity phase along c axis

- Phase difference evolves with time linearly in accordance with the ac Josephson relation.
- Static part P_l^s: π phase kink



dc supercurrent

- IV characteristics

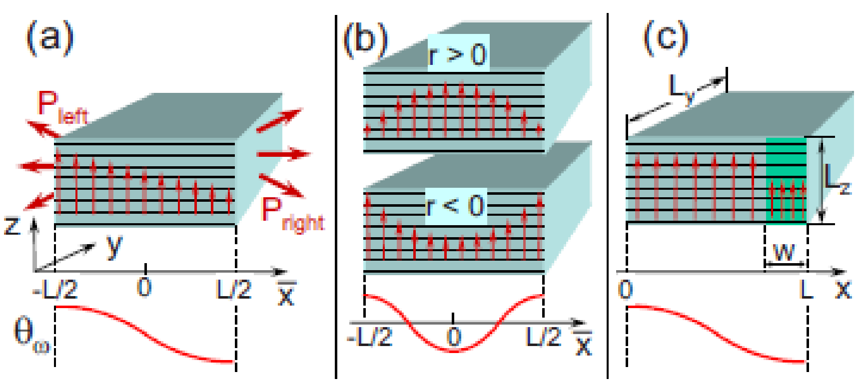
$$g_{10}^{m}(x) = \cos\frac{\pi x}{L}$$

$$J_{\text{ext}} = \beta\omega - \frac{\sin\varphi}{L}\int_0^L J_{-1}(Ag_{10}^{m})\cos P^s dx = \beta\omega\left(1 + A^2/4\right) \quad (2)$$

  - the kink P^s permits pumping large current and energy

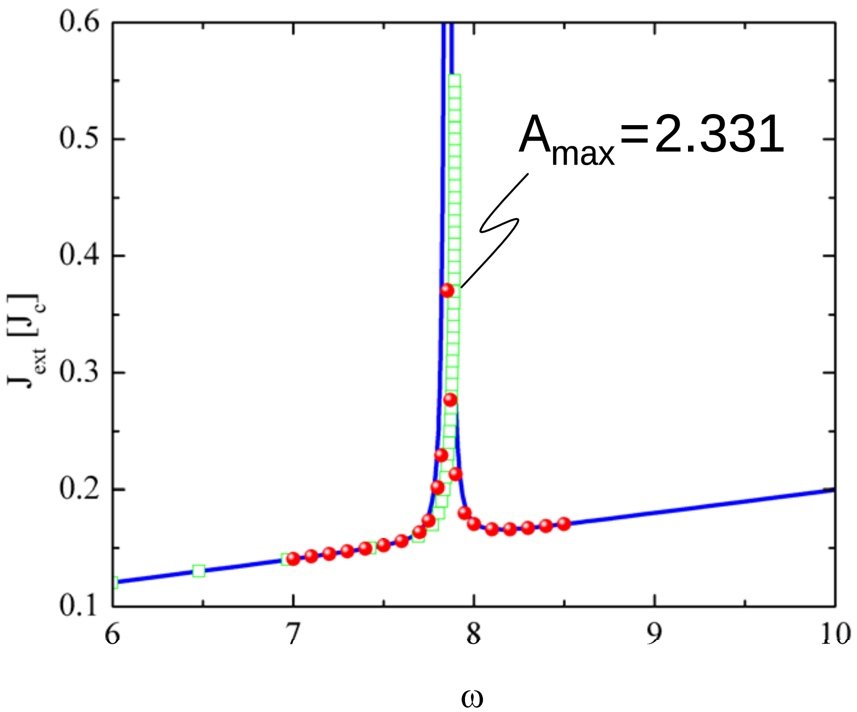
- Coupling between dc driving and transverse plasma

  - cf. Koshelev and Bualevskii



The new kink state builds up strong coupling automatically.

IV characteristics: current step at cavity voltage



- Cavity mode $c' = c/\sqrt{\varepsilon_c}$

$$\omega_{cavity} = \frac{\pi}{L_x} = 7.854 \Leftrightarrow \sim 0.6\mathrm{THz}$$

$$V = \phi_0 / 2\sqrt{\varepsilon_c} L_x = 1.225[\mathrm{mV}]$$

- ○ ac Josephson relation holds
- ○ negative differential resistance

cf. ZFS in single junction

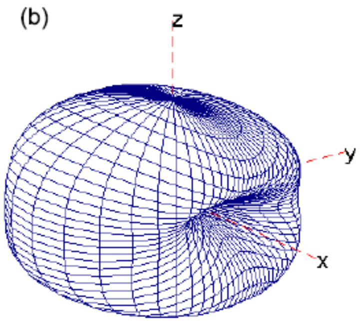
- Small A approximation: take linear terms in (3) & (4)

$$J_{ext} = \beta\omega\left[1 + \frac{\left(I_{10}^m\right)^2}{\left(\omega^2 - (\pi/L)^2\right)^2 + (\beta\omega)^2}\right]$$

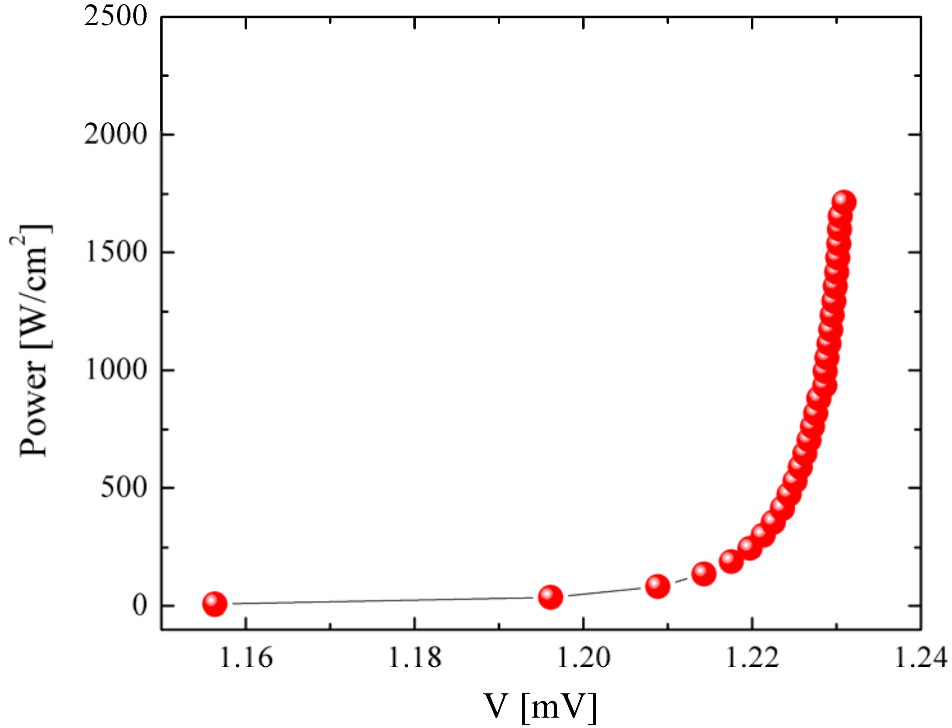
A. Koshelev, PRB **78**, 174509 (2008)

$$I_{10}^m = \frac{1}{L_x}\int_0^{L_x} g_{10}^m \cos P^s dx \approx 2/\pi$$

Radiation power: simulation



- Pyonting vector at the junction edges



- parameters

$L_x = 80\mu m$

$\varsigma = 4.44 \times 10^5$

$\beta = 0.02$

$\varepsilon_c = 10$

$|Z| = 1000$

$\theta = \pi/4$

- Radiated energy: taking the mesa in the experiments

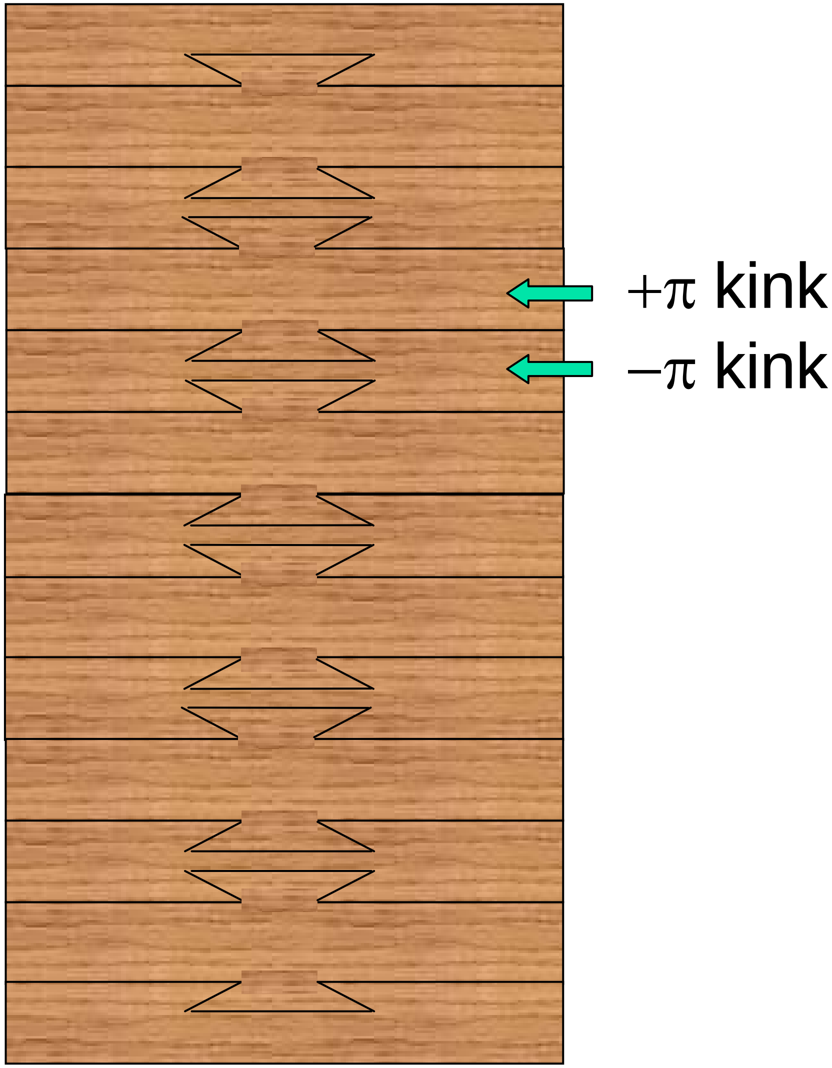
$P = 2000 \times L_z \times L_y = 6$ mW

- optimal value upon Z variation

Metaphor

Zigzag Staircase

Wedged Blocks

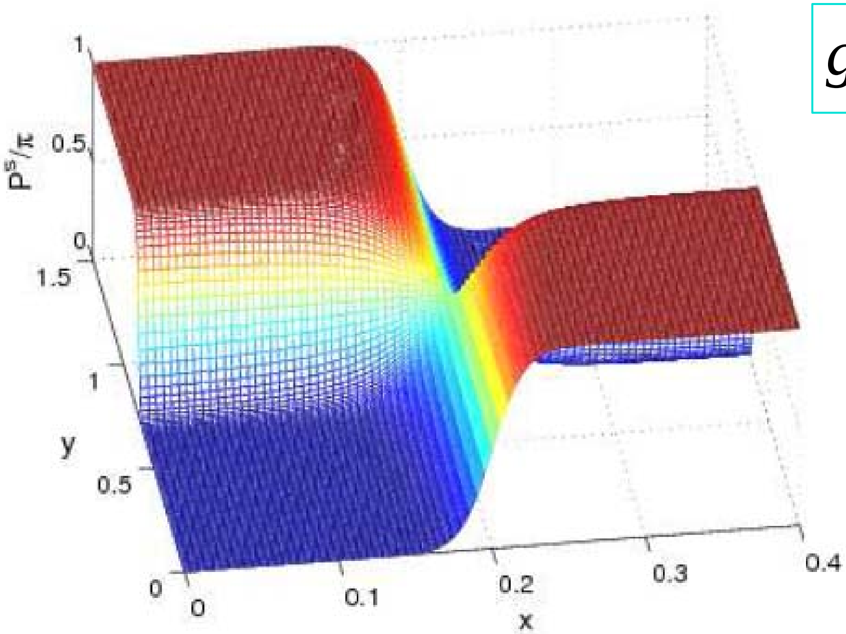


Kink state: (1,1) mode of rectangular mesa

- Solution:

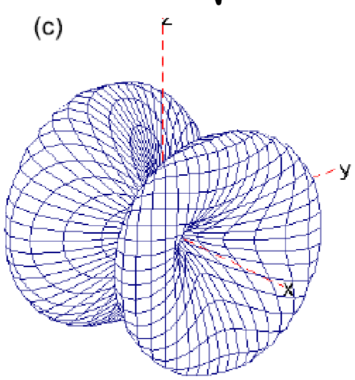
$$P_l(x,y,t)=\omega t+A\cos\frac{\pi x}{L_x}\cos\frac{\pi y}{L_y}\sin(\omega t+\varphi)+f_l P^s(x,y)$$

- 2D phase kink: $\left(\partial_x^2+\partial_y^2\right)P^s=q\varsigma\cos\varphi J_{-1}(Ag_{11}^m)\sin P^s$



$$g_{11}^m(x,y)=\cos(\pi x/L_x)\cos(\pi y/L_y)$$

$$\omega_{11}=\sqrt{(\pi/L_x)^2+(\pi/L_y)^2}$$



K. Kadowaki et al. Physica C 468, 634 (2008); and preprints

Cylindrical geometry

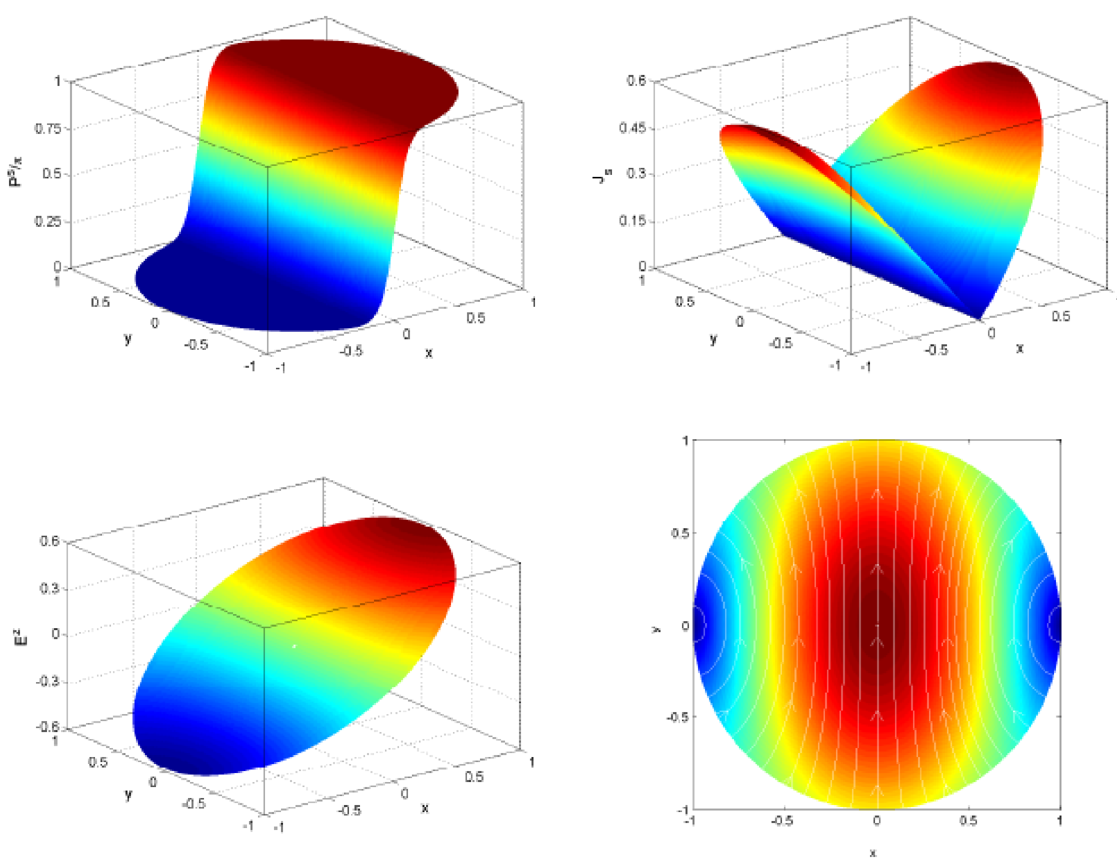
XH and S.-Z. Lin, PRB **78**, 134510 (2008)
Kadowaki group: conf. talks; preprint; poster
XH and S.-Z. Lin, in preparation

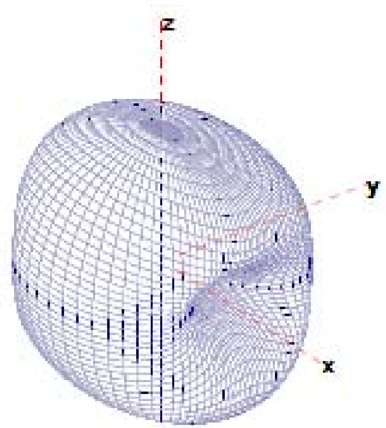
$$P_l(\boldsymbol{r},t) = \omega t + A J_1\left(\frac{v_{11}^{c}}{a}\rho\right)\cos\phi\sin(\omega t+\varphi) + f_l P^s(\boldsymbol{r})$$

$$f_{11}^{c} = 1.8412/a$$

$$E^z(\mathbf{r},t) = \partial \tilde{P} / \partial t$$

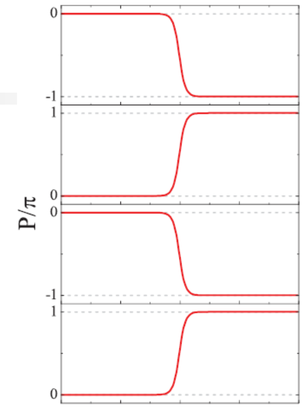
$$\mathbf{B}(\mathbf{r},t) = -\nabla \times \left(\tilde{P}(\mathbf{r},t)\mathbf{z}\right)$$





Summary

Thank you !



- We found a new dynamic state in Josephson junctions
  - π kink in superconductivity phase
- This is a device converting large dc energy into THz radiation.
  - the basic principle is the Josephson effect.
  - feeding grass to get milk,   pumping water to get oil
  - efficiency<10%, dissipations inevitable, no free lunch !
- Our theory explains all the important features of experiments so far.
- The theoretical picture is based on large-scale computer simulations
  - intuition very limited for highly nonlinear sine-Gordon systems