

# Competition and Coexistence between Néel order and $d$ -wave Singlet RVB

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## Abstract

The coexistence of antiferromagnetism and  $d$ -wave superconductivity is known to be realized in the two-dimensional  $t$ - $J$  model on the square lattice. It is shown in this paper that this coexistence is not a general feature and is significantly suppressed by  $t''$ , the third nearest-neighbor hopping. This effect of  $t''$  is argued on the material dependence of '1/8 anomalies' in high- $T_c$  cuprates.

*Key words:* antiferromagnetism, superconductivity, coexistence,  $t$ - $J$  model, cuprates

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## 1. Introduction

The coexistence of antiferromagnetism (AF) and  $d$ -wave superconductivity ( $d$ SC) is known to be realized in the two-dimensional (2D)  $t$ - $J$  model[1–3] and extended Hubbard model[4,5]. Since these models are believed to be minimal for high- $T_c$  cuprates, it seems that the coexistence might be a general feature in high- $T_c$  cuprates. However, as we report in this paper, such coexistence is controlled by the long-range hopping integral  $t''(> 0)$ , and is significantly suppressed with a moderate value of  $t''$ .

## 2. Model and Formalism

We take the 2D  $t$ - $J$  model on the square lattice:

$$H = - \sum_{i,j,\sigma} t^{(l)} \tilde{c}_{i\sigma}^\dagger \tilde{c}_{j\sigma} + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

defined in the Fock space with no doubly occupied sites. The  $\tilde{c}_{i\sigma}$  ( $\mathbf{S}_i$ ) is an electron (a spin) operator. The hopping integrals,  $t^{(l)}$ , are assumed between the  $l$ th ( $l \leq 3$ )

nearest-neighbor (n.n.) sites, and we denote  $t^{(1)} = t$ ,  $t^{(2)} = t'$ , and  $t^{(3)} = t''$ . The  $J (> 0)$  is superexchange coupling between the n.n. spins. We adopt the slave-boson formalism and introduce the slave particles as  $\tilde{c}_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i$ , where  $f_{i\sigma}$  ( $b_i$ ) is a fermion (boson) operator. The local constraint is described by  $\sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1$  at every site  $i$ .

To investigate the interplay between AF and  $d$ SC, we analyze this model by introducing the following mean fields: the resonating valence bond (RVB),  $\chi^{(l)} \equiv \langle \sum_{\sigma} f_{i\sigma}^\dagger f_{j\sigma} \rangle$ ,  $\langle b_i^\dagger b_j \rangle$  and  $\Delta_\tau \equiv \langle f_{i\uparrow} f_{i+\tau\downarrow} - f_{i\downarrow} f_{i+\tau\uparrow} \rangle$  with  $\tau = x, y$ , and the AF,  $m \equiv \frac{1}{2} \langle \sum_{\sigma} \sigma f_{i\sigma}^\dagger f_{i\sigma} \rangle e^{i\mathbf{Q} \cdot \mathbf{r}_i}$  with  $\mathbf{Q} = (\pi, \pi)$ . These mean fields are taken to be real constants independent of lattice coordinate  $i$ . Assuming the boson to be condensed at the bottom of its band and loosing the local constraint to the global one, we determine mean fields self-consistently.

## 3. Results

Figure 1(a) shows the phase diagram on the plane of temperature versus hole density for the band parameter,  $t/J = 4$ ,  $t'/t = -1/6$  and  $t''/t = 0$ , which will be appropriate to LSCO. The  $T_N$  and  $T_{RVB}^{AF}$  are the onset

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istence is implied in LSCO with hole density around 1/8[7], known as one of the ‘1/8 anomalies’. Similar anomaly is reported also in YBCO[8] and Bi2212[9] in the  $\mu$ SR experiments, which is, however, sharply different from the case of LSCO in that the precession of the muon spin is not observed and the Zn-doping is necessary. Since the  $\mu$ SR data are taken in the  $d$ SC state, this material dependence of ‘1/8 anomalies’ may indicate that the coexistence with AF is less favored in YBCO and Bi2212 than in LSCO. From the previous studies, the existence of moderate value of  $t'' (> 0)$  is expected in YBCO and Bi2212, and not in LSCO. Therefore, the material dependence of ‘1/8 anomalies’ will be understood by the present effect of  $t''$ , namely an intrinsic effect of the electronic system.

## 5. Summary

In summary, we have found that possible coexistence of AF and  $d$ -RVB is controlled by  $t'' (> 0)$ . The material dependence of ‘1/8 anomalies’ will result from this significant effect of  $t''$ .

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