

# Griffiths wings associated with electronic nematic transition

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**Abstract.** A spatial  $xy$  anisotropy generated by a uniaxial pressure and strain can control a phase transition. In particular, the  $xy$  anisotropy couples directly to electronic nematic order and thus is expected to play a crucial role in a system exhibiting nematicity. Here we study the phase diagram of an electronic nematic phase transition by introducing the  $xy$  anisotropy. In a mean-field theory, we find that a first order transition occurs near van Hove filling and its phase boundary forms a wing structure, which we term a Griffiths wing. The anisotropy of the electronic system exhibits a discontinuous change by crossing the wing, leading to a meta-nematic transition, i.e., the analog to a meta-magnetic transition observed in ferromagnetic systems such as  $\text{UGe}_2$  and  $\text{UCoAl}$ . However, in contrast to ferromagnetic systems, the upper edge of the wing shows a non-monotonic temperature dependence. The Griffiths wing interpolates between a two-dimensional system and a nearly one-dimensional system by changing the  $xy$  anisotropy. We also study the effect of nematic order-parameter fluctuations in a functional renormalization-group scheme. We find that the Griffiths wing is very sensitive to the fluctuations and is easily broken into two pieces, leading to a phase diagram topologically different from the mean-field result. We discuss that the Griffiths wing associated with nematicity can be related to various materials such as high- $T_c$  cuprates, bilayer ruthenates, quasi-one-dimensional metals, and cold atom systems.

## 1. Introduction

Electronic nematic states are observed in a number of interacting electron systems: Two-dimensional electron gases [1, 2], high-temperature superconductors of cuprates [3, 4] and pnictides [5], the bilayer strontium ruthenate  $\text{Sr}_3\text{Ru}_2\text{O}_7$  [6], and an actinide material  $\text{URu}_2\text{Si}_2$  [7]. In the nematic states the orientational symmetry is broken, but the other symmetries are retained. Depending on electronic degrees of freedom responsible for nematicity, we may define three kinds of nematicity: spin, orbital, and charge nematicity. The spin nematicity is driven by frustration between magnetic interactions [8]. The orbital nematic state is a kind of an orbital order, for example, a spontaneous occupation difference between  $d_{yz}$  and  $d_{zx}$  orbitals in a  $d$ -electron system [9, 10]. There are two routes toward a charge nematic state, via melting of spin-charge stripes [11] and a  $d$ -wave Pomeranchuk instability ( $d$ PI) of a normal phase [12, 13, 14].

Materials often have  $xy$  anisotropy coming from the crystal structure due to orthorhombicity. The anisotropy can also be applied externally by a uniaxial pressure and strain. Since the electronic nematic order couples directly to the  $xy$  anisotropy, the anisotropy is expected to play a crucial role in a system exhibiting nematicity. In fact, it was shown that the anisotropy

introduced to the system is strongly enhanced by the underlying nematic correlations [13, 15, 16]. This enhancement was discussed to explain the strong anisotropy of magnetic excitation spectra [17, 18] and of Nernst coefficient [19] observed in high-temperature superconductors [20, 21, 22, 23]. Except for these studies a role of anisotropy has not been well addressed.

In this paper, we study an electronic nematic phase transition in the presence of  $xy$  anisotropy. In particular, we focus on the nematic instability driven by the  $d$ PI. The  $d$ PI is known to exhibit a dome-shaped phase boundary around van Hove filling in the plane spanned by the chemical potential and temperature [24, 25]. The edges of the dome correspond to the lines of a first order phase transition and the rest is a second order transition. The end points of the second order line are tricritical points.

The presence of a tricritical point (TCP) implies a wing structure when a conjugate field to the corresponding order parameter is applied to the system. This was first shown for He<sup>3</sup>-He<sup>4</sup> mixtures by Griffiths [26]. However the wing structure predicted by Griffiths, which we term the Griffiths wing, was not tested for He<sup>3</sup>-He<sup>4</sup> mixtures because the conjugate field to the superfluid order parameter is not accessible in experiments. On the other hand, it was found theoretically that a TCP is in general present in itinerant ferromagnetic systems [27]. The order parameter is magnetization and its conjugate field is a magnetic field in that case. Recently, the Griffiths wing was observed clearly in UGe<sub>2</sub> [28] and UCoAl [29] by applying a magnetic field. The Griffiths wing corresponds to a metamagnetic transition, that is, the system exhibits a jump of the magnetization when crossing the wing.

Here we report a Griffiths wing associated with the electronic nematic phase transition driven by the  $d$ PI in a two-dimensional system. A field conjugate to the nematic order parameter is  $xy$  anisotropy. We find that the Griffiths wing actually emerges, but in contrast to previous studies [26, 27], the upper edge of the wing shows a non-monotonic temperature dependence. The Griffiths wing is found to extend to a nearly one-dimensional system by increasing the anisotropy. The Griffiths wing is, however, very sensitive to fluctuations of the nematic order parameter and is easily broken up into two pieces. We discuss that the concept of the Griffiths wing can be relevant to a variety of physical systems. The present paper is a summary of a full detailed study [30] and will serve to catch essential points quickly.

## 2. Model

We employ a one-band model on a square lattice and consider a situation where a nematic tendency becomes dominate at low energy over the other ordering tendencies such as superconductivity and charge/spin density waves. We also apply  $xy$  anisotropy to the system. These situations may be described by the following Hamiltonian,

$$H = \sum_{\mathbf{k}, \sigma} (\epsilon_{\mathbf{k}}^0 - \mu) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} - \frac{1}{2N} \sum_{\mathbf{q}} g(\mathbf{q}) n_d(\mathbf{q}) n_d(-\mathbf{q}) - \mu_d n_d(\mathbf{0}), \quad (1)$$

where  $c_{\mathbf{k}\sigma}^\dagger$  ( $c_{\mathbf{k}\sigma}$ ) is the creation (annihilation) operator of electrons with momentum  $\mathbf{k}$  and spin  $\sigma$ ,  $\mu$  is the chemical potential, and  $N$  is the number of sites. The first term describes the kinetic energy of electrons and  $\epsilon_{\mathbf{k}}^0$  is a usual tight binding dispersion on a square lattice, i.e.,  $\epsilon_{\mathbf{k}}^0 = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$ . The second term is an effective interaction driving a  $d$ PI. This effective interaction is obtained from microscopic models such the  $t$ - $J$  [12, 13], Hubbard [14, 31], and general models with central forces [32]. We assume that the coupling strength  $g(\mathbf{q})$  has a peak at  $\mathbf{q} = 0$ , that is, forward scattering dominates.  $n_d(\mathbf{q})$  is a  $d$ -wave weighted density:  $n_d(\mathbf{q}) = \sum_{\mathbf{k}, \sigma} d_{\mathbf{k}} c_{\mathbf{k}-\frac{\mathbf{q}}{2}\sigma}^\dagger c_{\mathbf{k}+\frac{\mathbf{q}}{2}\sigma}$  with a  $d$ -wave form factor  $d_{\mathbf{k}} = \cos k_x - \cos k_y$ . A new aspect of the present model lies in the third term in Hamiltonian (1). This term can be absorbed in the kinetic term as

$$\epsilon_{\mathbf{k}} = -2(t_x \cos k_x + t_y \cos k_y) - 4t' \cos k_x \cos k_y, \quad (2)$$

where  $t_x = t(1 + \mu_d/2t)$  and  $t_y = t(1 - \mu_d/2t)$ . The third term, therefore, introduces  $xy$  anisotropy to the system and models the effect of a uniaxial pressure, strain, and orthorhombicity due to the underlying crystal structure. Since the presence of  $t'$  is of minor importance in the present study, we put  $t' = 0$  hereafter.

The introduction of  $\mu_d$  in Hamiltonian (1) is responsible for the emergence of the Griffiths wing and is the most crucial point of the present study. By adding the third axis of  $\mu_d$ , we will determine a phase diagram of the nematic transition. Because of the reflection symmetry with respect to the axes of  $\mu = 0$  and  $\mu_d = 0$ , we focus on a region  $\mu > 0$  and  $\mu_d > 0$ . Since the nematic order parameter exhibits a jump by crossing a Griffiths wing, we determine a phase boundary of a first order transition in three-dimensional space spanned by  $\mu$ ,  $\mu_d$ , and  $T$ . In particular, the upper edge of the wing corresponds to a critical end line (CEL), where a first order transition vanishes. The CEL is determined by the condition

$$\frac{\partial\omega}{\partial\phi} = \frac{\partial^2\omega}{\partial\phi^2} = \frac{\partial^3\omega}{\partial\phi^3} = 0, \quad (3)$$

where  $\omega$  is the free energy and  $\phi$  is a nematic order parameter.

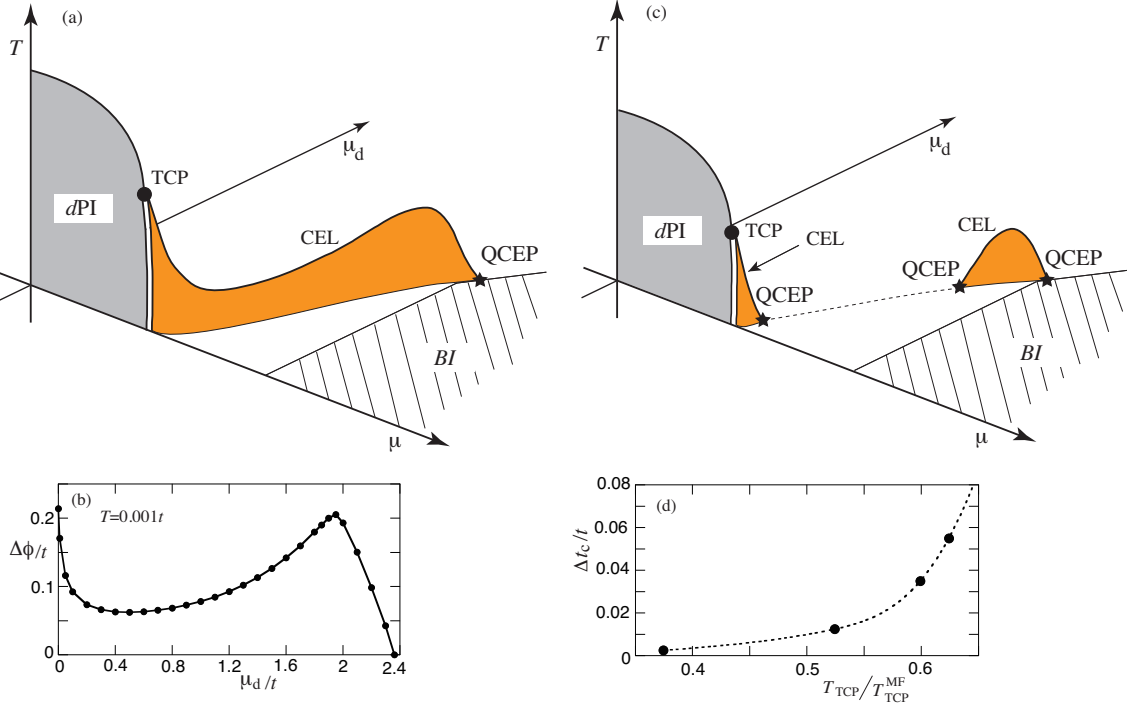
### 3. Results

We first study Hamiltonian (1) in a mean-field approximation. We decouple the interaction term by introducing the nematic order parameter

$$\phi = gn_d(\mathbf{0})/N, \quad (4)$$

where  $g = g(\mathbf{0}) > 0$ . It is straightforward to compute the free energy. We then solve Eq. (3) numerically to determine the upper edge of the Griffiths wing. Besides the edge, the whole structure of the wing is determined by searching the chemical potential where a first order transition occurs for a given  $\mu_d$  and  $T$ . The obtained phase diagram is shown in Fig. 1(a) schematically. At zero anisotropy ( $\mu_d = 0$ ) a dPI occurs around van Hove filling, from which the chemical potential is measured. The transition is second order at high  $T$  (solid line) and changes to a first order transition at low  $T$  (double line) through a TCP (solid circle), as already obtained in literature [24, 25]. With increasing  $\mu$ , the band is eventually filled up and the band insulating (BI) state is realized in the striped region. The Griffiths wing (colored in orange) emerges from the first order line and extends to a region of a finite  $\mu_d$ . It stands almost vertically on the plane of  $\mu$  and  $\mu_d$ , and evolve close to van Hove filling on that plane. In contrast to the Griffiths wing found in  $\text{He}^3\text{-He}^4$  mixtures [26] and ferromagnetic systems [27, 28, 29], the CEL exhibits a non-monotonic temperature dependence. It is rapidly suppressed by applying the anisotropy  $\mu_d$ , but it recovers to form a broad peak around  $\mu_d = 2$ . The CEL eventually vanishes at the boarder of the BI phase, leading to a quantum critical end point (QCEP) there. When crossing the wing, the system exhibits a jump of nematicity, namely a meta-nematic transition. The jump of the nematic order parameter is plotted along the bottom of the wing in Fig. 1(b). It is interesting that  $\Delta\phi$  around  $\mu_d = 2$  is comparable to that at  $\mu_d = 0$  although the system is strongly anisotropic due to the presence of a large  $\mu_d$ .

These mean-field results can be easily understood by considering the evolution of the band structure. The bare band dispersion  $\epsilon_{\mathbf{k}}$  [see Eq. (2)] has saddle points at  $(\pi, 0)$  and  $(0, \pi)$  as long as  $\mu_d < 2t$ . For  $\mu_d > 2t$ , however, they change to  $(0, 0)$  and  $(\pi, \pi)$ . As a result, the band becomes very flat around  $\mu_d = 2t$ , leading to a pronounced enhancement of the density of states. This is the reason why both the CEL and  $\Delta\phi$  exhibit a peak around  $\mu_d = 2t$ . The value of  $\mu_d = 2t$  indicates that the system becomes almost one-dimensional. Hence the Griffiths wing associated with nematicity interpolates between a two- and nearly one-dimensional system by controlling the anisotropy  $\mu_d$ .



**Figure 1.** (Color online) (a) Schematic phase diagram obtained in a mean-field approximation. (b) Jump of the nematic order parameter across the wing at  $T = 0.001t$ . (c) Schematic phase diagram in the presence of nematic order-parameter fluctuations. (d) Critical anisotropy of the hopping integral,  $\Delta t_c/t$ , to obtain the QCEP as a function of the ratio of the tricritical temperature and its mean-field value.

The mean-field results are, however, very sensitive to fluctuations of the nematic order parameter. In the case of  $\mu_d = 0$ , Refs. [33, 34] addressed the effect of nematic order-parameter fluctuations on the mean-field results in a functional renormalization-group scheme. We extend such an analysis to the presence of anisotropy ( $\mu_d \neq 0$ ) and determine a first order transition associated with the Griffiths wing as well as its upper edge [Eq. (3)] numerically. Obtained results are shown in Fig. 1(c) schematically. Applying the anisotropy  $\mu_d$ , the CEL rapidly drops to zero, leading to a tiny wing terminating at a QCEP. We then have a crossover region depicted by the dashed line, where the order parameter shows a rapid change, but without a jump. With further increasing  $\mu_d$ , another wing emerges with two QCEPs. While the phase diagram at  $\mu_d = 0$  is essentially the same as the mean-field result [Fig. 1(a)], the phase diagram at  $\mu_d \neq 0$  becomes qualitatively different from the mean-field results, suggesting a dramatic effect of order-parameter fluctuations once the anisotropy is introduced. To quantify the effect of the fluctuations, we consider the critical external anisotropy to obtain a QCEP, which we express in terms of  $\Delta t_c/t = \frac{t_x - t_y}{t_x + t_y} = \mu_d/2t$ . In Fig. 1(d),  $\Delta t_c/t$  is plotted as a function of the ratio of the tricritical temperature and its mean-field value ( $T_{TCP}/T_{TCP}^{MF}$ ). We see that when  $T_{TCP}$  is suppressed by fluctuations, for example, by half, a very small anisotropy ( $\Delta t_c/t \approx 0.01$ ) is sufficient to yield a QCEP. Furthermore the strength of fluctuations we consider is weak in the sense that the mean-field phase diagram at  $\mu_d = 0$  is qualitatively intact even in the presence of the fluctuations. Therefore the Griffiths wing is very sensitive to nematic order-parameter fluctuations and a QCEP can be easily obtained with a weak anisotropy.

#### 4. Conclusion

We have clarified the nematic phase diagram of the  $d$ PI in the presence of  $xy$  anisotropy. The  $xy$  anisotropy is a field conjugate to the nematic order parameter and thus is expected to play a crucial role. We have found that the Griffiths wing emerges from a first order transition line and interpolates between a two- and nearly one-dimensional system by changing the anisotropy. In a mean-field theory, the upper edge of the wing, namely the CEL, exhibits a non-monotonic temperature dependence, in sharp contrast to the original work by Griffiths [26] and magnetic systems [27, 28, 29]. Furthermore, we have found that the wing is very sensitive to order-parameter fluctuations and is easily broken up into two pieces. A tiny wing near zero anisotropy makes the system close to a QCEP even though the transition is of first order at zero anisotropy. The other wing is realized for a very strong anisotropy. Although the two wings might seem to be completely separated by the fluctuation effect, they are connected by a crossover line [dashed line in Fig. 1(c)] as reminiscence of a single Griffiths wing in the absence of fluctuations [Fig. 1(a)].

Our Hamiltonian (1) is a low-energy effective one-band model and addresses the nematic transition in a rather general setup, independent of microscopic details. Furthermore the nematic interaction is obtained from various microscopic models [12, 13, 14, 31, 32, 35]. Hence we may apply the present theory to a variety of materials exhibiting a nematic tendency. Since there always exist fluctuations of the order parameter in real materials, Fig. 1(c) may be more realistic than Fig. 1(a). Thus we bear Fig. 1(c) in mind and discuss briefly implications for several materials.

For cuprate superconductors, neutron scattering experiments showed that the magnetic excitation spectrum becomes anisotropic in momentum space. The relatively weak anisotropy was observed for  $\text{YBa}_2\text{Cu}_3\text{O}_{6.85}$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$  [20, 21], but the anisotropy seems to be pronounced suddenly by crossing the oxygen concentration around 6.45 [22]. Since superconducting samples of Y-based cuprates have an intrinsic  $xy$  anisotropy coming from the CuO chain structure, Y-based cuprates are located along the axis of a small  $\mu_d$  ( $\approx 0.03 - 0.04$  [17]) in Fig. 1(c). With decreasing  $\mu$  (hole picture), namely decreasing the oxygen concentration, the system may cross the tiny wing or pass close to the QCEP in Fig. 1(c), which may explain a sudden change of the anisotropy observed in the magnetic excitation spectrum [20, 21, 22].

The bilayer ruthenate  $\text{Sr}_3\text{Ru}_2\text{O}_7$  is a tetragonal system ( $\mu_d = 0$ ) and exhibits an electronic nematic instability [6]. One of bands crossing the Fermi energy forms a two-dimensional Fermi surface, which is located very close to saddle points and contributes to the large density of states. Focusing on such a band,  $\text{Sr}_3\text{Ru}_2\text{O}_7$  is often studied in terms of one-band models with an interaction of our second term in Hamiltonian (1) [36, 37, 38, 39, 40, 41, 42]. In particular, one-band theory turns out to capture major aspects of the experimental phase diagram [6] as shown in Figs. 1(a) and (c) at  $\mu_d = 0$  (see Ref. [43] for details). The present theory then predicts that the CEL rapidly drops by applying a strain along the  $x$  or  $y$  direction, possibly reaching a QCEP in experiments. While an  $xy$  anisotropy is also generated by introducing a magnetic field along the  $x$  or  $y$  direction [6], the coupling to electrons is different from our anisotropic field of  $\mu_d$ .

A broken Griffiths wing is realized for a strong anisotropy in Fig. 1(c). Such a strong anisotropy is easily realized in quasi-one-dimensional metals. While these systems already have strong anisotropy, our theory predicts that the anisotropy of the electronic system can change drastically by crossing the wing, for example, by controlling carrier density or applying a uniaxial pressure when the system is located near van Hove filling. Although we are not aware of experimental papers discussing such a sudden change of anisotropy, there are theoretical works reporting it in a different context [44, 45]. Since  $xy$  anisotropy of physical quantities in a strongly anisotropic system was not likely recognized as something related to nematicity, we may reasonably wait for further experiments.

Condensed matter systems have their own intrinsic anisotropy given by the underlying crystal structure and thus  $\mu_d$  cannot be changed much externally away from the intrinsic anisotropy. However, the anisotropy is fully tunable in optical lattices in a cold atom system by changing the strength of laser beams between the  $x$  and  $y$  direction [46]. Given that dipolar interaction can generate the attractive interaction of the  $d$ PI [35], our predicted structure of the Griffiths wing [Fig. 1(c)] may be tested by employing dipolar fermion gases.

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