

1 Introduction

The program calculates the transmission in the L-C or C-R interface. The system the program can calculate is written in the top part of Fig.1. The effects of semi-infinite electrodes, L or R, is calculated using the surface Green function, G_L or G_R . Then, the program takes into account of The effects electrode to the C region via self-energy Σ_L and Σ_R . In this way, the program solves the system shown in the bottom of Fig.1 which is equivalent to the top of the figure. The program can also apply the source-drain bias between L and R.

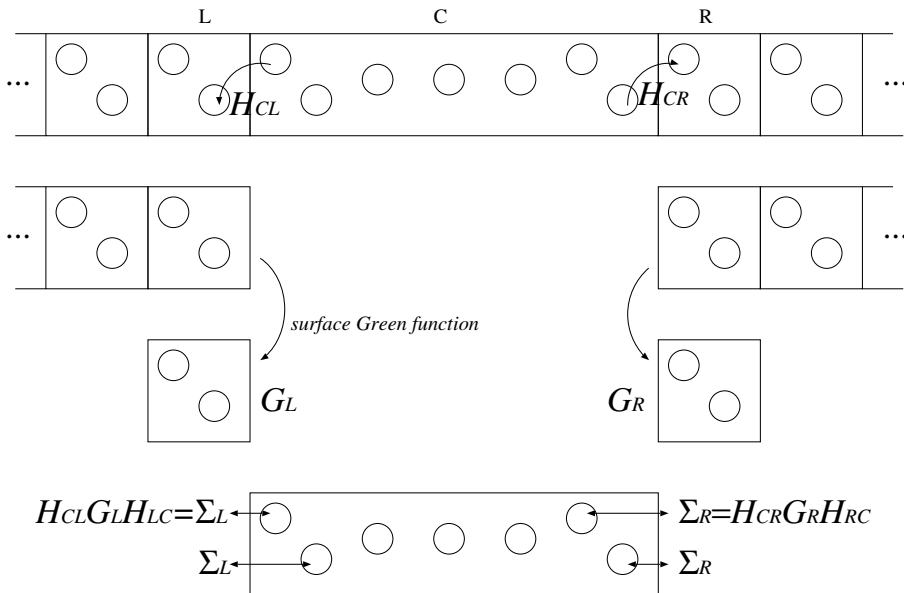


Figure 1:

2 Surface Green functions

What is necessary to calculate is Hamiltonian and overlap matrixes of electrodes, L and R. They are $H_{00_e[side]}$, $S_{00_e[side]}$, $H_{01_e[side]}$ and $S_{01_e[side]}$, side=0 for L and side=1 for R.

For L, $H_{01_e[side]}$ and $S_{01_e[side]}$ are transfer matrix to the left of the unit cell. For R, $H_{01_e[side]}$ and $S_{01_e[side]}$ are transfer matrix to the right of the unit cell. See also Fig.2.

The calculated Green functions are G_L and G_R .

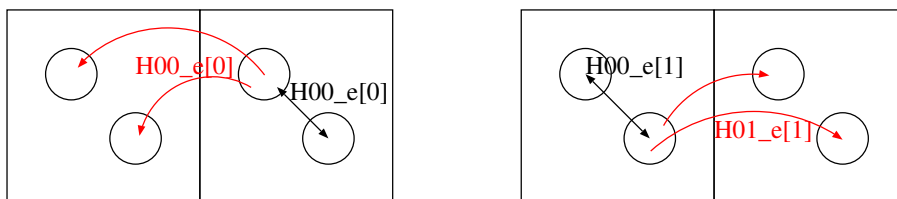


Figure 2:

3 Green function in the central region

What is necessary to calculate is Hamiltonian and overlap matrixes of the central region and to the electrodes, L and R. They are HCC, SCC, HCL, SCL and HCR and SCR. And the surface Green functions of the electrodes, G_L and G_R are necessary, of course.

The Green function of the central region, G , is

$$G(\omega) = \{\omega S_{CC} - H_{CC} - \Sigma_R(\omega) - \Sigma_L(\omega)\}^{-1} \quad (1)$$

$$\Sigma_L(\omega) = H_{CL}G_L(\omega)H_{LC} \quad (2)$$

$$\Sigma_R(\omega) = H_{CR}G_R(\omega)H_{RC} \quad (3)$$

The Lesser Green function of the central region is

$$G^< = G^R \Sigma^< G^A \simeq G^R (-\Sigma_L^R - \Sigma_L^A) f(\omega - \mu_L) - (\Sigma_R^R - \Sigma_R^A) f(\omega - \mu_R) G^A$$

where $G^R(\omega) = G(\omega + i\delta)$ and $G^A(\omega) = G(\omega - i\delta)$.

The Density matrix is evaluated as

$$\rho_{ij} = \text{Im} \int_{-\infty}^{E_1} d\omega \left(\frac{-1}{\pi} \right) G^R(\omega) + \int_{E_1}^{\infty} d\omega \frac{1}{2\pi i} (G_{ij}^<(\omega))$$

Analytic continuation is possible for the former integral.

$$\text{Im} \int_{-\infty}^{E_1} d\omega \left(\frac{-1}{\pi} \right) G^R(\omega) = \text{Im} \int_C^{E_1} d\omega \left(\frac{-1}{\pi} \right) G(\omega)$$

The final expression is

$$\rho_{ij} = \text{Im} \int_C^{E_1} d\omega \left(\frac{-1}{\pi} \right) G(\omega) + \int_{E_1}^{\infty} d\omega \frac{1}{2\pi i} (G_{ij}^<(\omega))$$

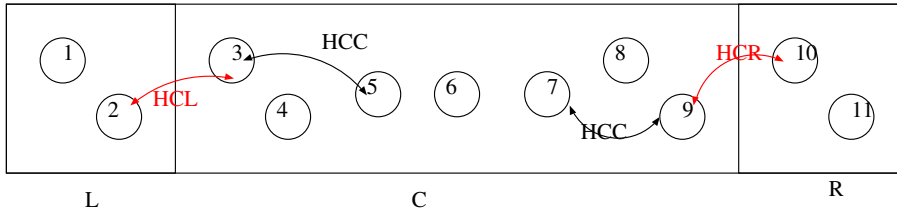


Figure 3:

4 Transmission

Once the Hamiltonian and overlap matrixes are solved. You can calculate transmission for any ω .

$$J = -\frac{e}{h} \int d\omega \text{Tr}[(f(\omega - \mu_L) - f(\omega - \mu_R))(\Gamma_L G_C^R \Gamma_R G_C^A)]$$

where $\Gamma_L = i(\Sigma_L^R - \Sigma_L^A)$ and $\Gamma_R = i(\Sigma_R^R - \Sigma_R^A)$

5 Overlapping region

5.1 $\rho_{ij} \rightarrow \rho(r)$

$$\rho(r) = \sum_{ij} \phi_i(r) \rho_{ij} \phi_j(r)$$

The secular equation is solved only in the C region. It means the indexes i and j for ρ_{ij} that you can solve is in the C region. There exists overlap between L and C region, however you can not include such effects. Thus you can not calculate $\rho(r)$ near the L region. Therefore the program employs the charge density $\rho(r)$ of the L region for such parts.

5.2 $\phi_i \rightarrow H$

The program define the potential, or the charge density, only in the C region. Thus, if the wavefunction ϕ_i or ϕ_j spans outside the C region, the program can not define the Hamiltonian matrix $\langle \phi_i | H | \phi_j \rangle$. Therefore the program use the Hamiltonian or overlap matrix of the L region for such parts.

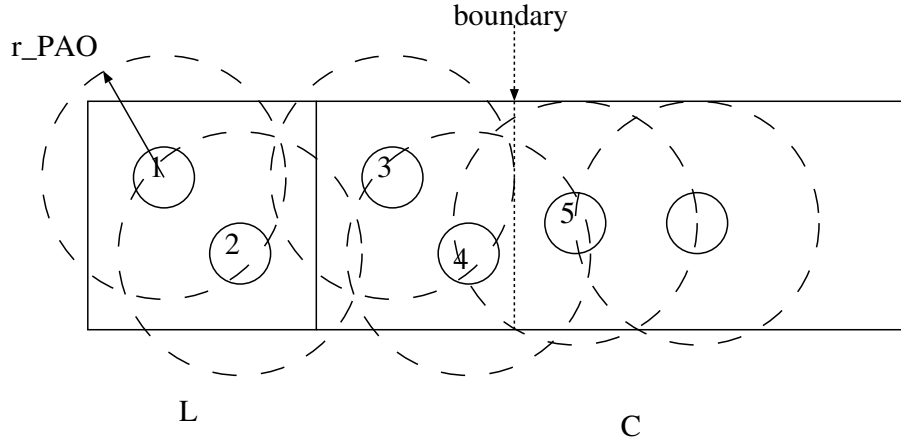


Figure 4: To evaluate $\rho(r)$ near the L region, the wavefunction ϕ_2 and the density matrix ρ_{2i} are necessary. Thus the program can not evaluate $\rho(r)$ whose r is left of the boundary. To evaluate $\langle \phi_3 | H | \phi_3 \rangle$, the potential in the L region is necessary.

6 Poisson equation

The program solves

$$\Delta V_H(r) = -4\pi\rho(r)$$

It is solved in this way.

$$\Delta(V_C(r) + V_B(r)) = -4\pi(\rho(r) + 0) \quad (4)$$

$$\Delta V_C(r) = -4\pi\rho(r) \quad (5)$$

$$\Delta V_B(r) = 0 \quad (6)$$

$\Delta V_C(r) = -4\pi\rho(r)$ is solved via FFT as a periodic system. $\Delta V_B(r) = 0$ is solved with the boundary condition.

$V(r)$ are transformed to $V(G_{\parallel}, z)$. Now you have $V_C(G_{\parallel}, z_i)$ in the C region, $i = 1, 2, \dots, l$. The boundary condition is $V_B(G_{\parallel}, z_0)$ and $V_B(G_{\parallel}, z_{l+1})$. The correction to the $V_C(G_{\parallel}, z_i)$, $i = 0$ and $l + 1$, is $dV_H(G_{\parallel}, z_i) = V_B(G_{\parallel}, z_i) - V_C(G_{\parallel}, z_i)$.

6.1 $G_{\parallel} \neq 0$

$$V_B(r) = \int dG_{\parallel} e^{iG_{\parallel} r_{\parallel}} \left\{ dV_H(G_{\parallel}, z_{l+1}) \left(e^{-G_{\parallel}(z_{l+1}-z)} - e^{-G_{\parallel}(z_{l+1}-2z_0+z)} \right) + dV_H(G_{\parallel}, z_0) \left(e^{-G_{\parallel}(z-z_0)} - e^{-G_{\parallel}(2z_{l+1}-z_0-z)} \right) \right\} / \left(1 - e^{-2G_{\parallel}(z_{l+1}-z_0)} \right) \quad (7)$$

6.2 $G_{\parallel} = 0$

Here $G_0 = 0$.

$$\Delta V_B(G_0, z) = 0 \quad (8)$$

$$V_B(G_0, z) = a(z - z_0) + b \quad (9)$$

a and b is determined from

$$V_B(G_0, z_0) = dV_H(G_0, z_0) + b \quad (10)$$

$$V_B(G_0, z_{l+1}) = dV_H(G_0, z_{l+1}) + a(z_{l+1} - z_0) + b \quad (11)$$

A Interpolation

$\rho(r)$ and $V(r)$ are evaluated on the FFT grid. However, the FFT grid of the C region is different from that of the electrode, because the unit cell is different. The value must be interpolated.

An input data set

$$H((\gamma + n)/N_{in})$$

where $n = 0, N_{in} - 1$ and assuming the periodic boundary condition. The interpolated data set

$$H((\alpha + \beta m)/N_{out})$$

where m is integer and can be beyond the range of $[0:N_{out} - 1]$

Forward transformation

$$h(k) = \sum_{n=0, N_{in}-1} H(n + \gamma) \exp(-2\pi i k (n + \gamma)/N_{in}) \quad (12)$$

$$= \exp(-2\pi i k \gamma/N_{in}) \sum_{n=0, N_{in}-1} H(n + \gamma) \exp(-2\pi i k n/N_{in}) \quad (13)$$

$$= \exp(-2\pi i k \gamma/N_{in}) h^0(k) \quad (14)$$

where

$$h^0(k) = \sum_{n=0, N_{in}-1} H(n + \gamma) \exp(-2\pi i k n/N_{in})$$

can be calculated via FFT.

Backward transformation of $H((\alpha + \beta n)/N_{out})$.

$(\alpha + \beta n)/N_{out}$ can be beyond the range of $[0:1]$.

$$H'(\alpha + \beta n) = \sum_{k=0, N_{in}-1} h(k) \exp(2\pi i k (\alpha + \beta n)/N_{out}) \quad (15)$$

$$= \sum_{k=0, N_{in}-1} \exp(-2\pi i k \gamma/N_{in}) h^0(k) \exp(2\pi i k (\alpha + \beta n)/N_{out}) \quad (16)$$

$$= \sum_{k=0, N_{in}-1} h^0(k) \exp(2\pi i k ((\alpha + \beta n)/N_{out} - \gamma/N_{in})) \quad (17)$$

Note, the transformation above can not give all of $(\alpha + \beta n)/N_{out}$ correctly. In order to interpolate data, use the formula below

$$H(\alpha + \beta n) = \sum_{k=0, N_{in}/2} h^0(k) \exp(2\pi i k ((\alpha + \beta n)/N_{out} - \gamma/N_{in})) \quad (18)$$

$$+ \sum_{k=-N_{in}/2, -1} h^0(k) \exp(-2\pi i k ((\alpha + \beta n)/N_{out} - \gamma/N_{in})) \quad (19)$$

where $h^0(k - N_{in}) = h^0(k)$.

A Structure of the program

```

Input_std
  TRAN_Input_std
    TRAN_Set_SurfOverlap (read data of electrode calculated previously)
  TRAN_Input_std_Atoms
truncation
  TRAN_adjust_Ngrid
  TRAN_adjust_Grid_Origin
DFT
  TRAN_Set_Electrode_Grid (interpolate griddata of electrodes)
  TRAN_Allocate_Cregion
  scf_loop {
    Poisson | TRAN_Poisson (solve Poisson equation with the boundary condition)
    Cluster_DFT | Band_DFT | TRAN_DFT (calculate CDM from H and S)
    TRAN_Overwrite_Densitygrid (overwrite the griddata of electrodes)
  }
  TRAN_Output_Trans_HS (output H and S to calculate transmission later)
  TRAN_Deallocate_Cregion

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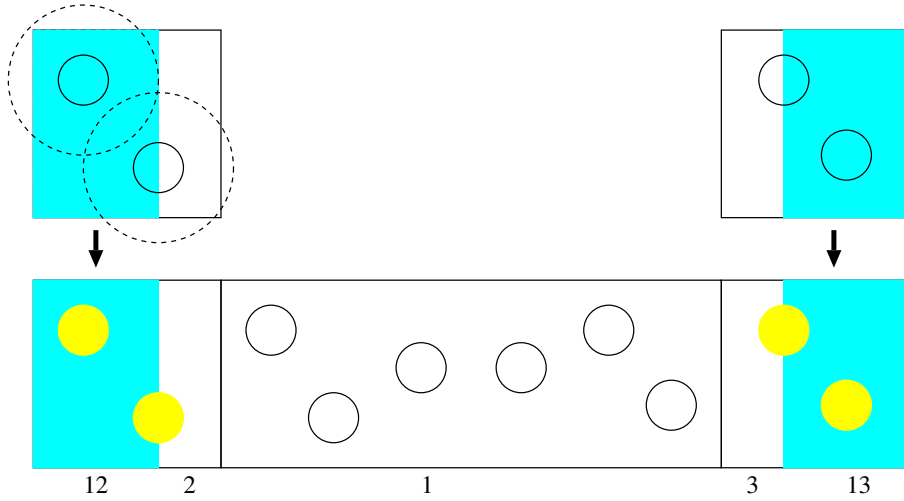


Figure 5: Electrode region = $[0:\text{TRAN_grid_bound}[0]]$ for left, $[\text{TRAN_grid_bound}[0], \text{Ngrid1}-1]$. They are shown in blue. $\text{TRAN_region}[1:\text{atomnum}] = 12$ (left, yellow), 2 (left, open), 1 (center), 3 (right, open), 13 (right, yellow).

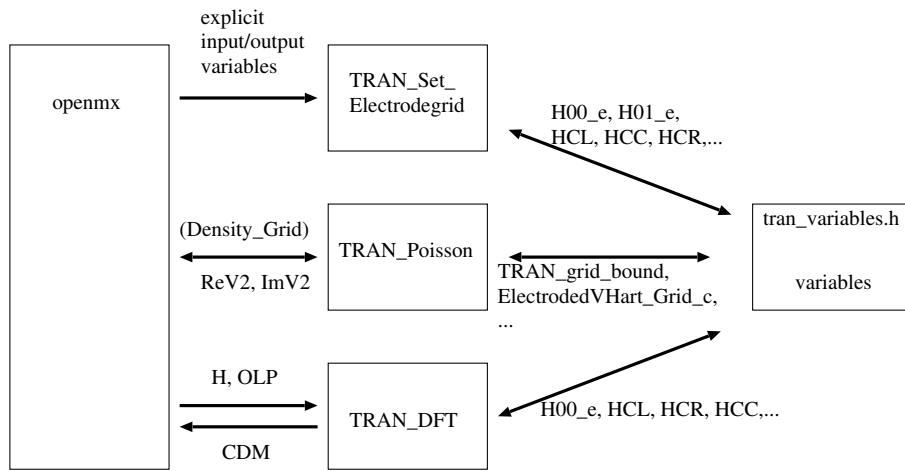


Figure 6: